AT6601 - Automotive Engine Components Design

UNIT I INTRODUCTION

Introduction

The knowledge of materials and their properties is of great significance for a design engineer. The machine elements should be made of such a material which has properties suitable for the conditions of operation. In addition to this, a design engineer must be familiar with the effects which the manufacturing processes and heat treatment have on the properties of the materials. In this chapter, we shall discuss the commonly used engineering materials and their properties in Machine Design.

Classification of Engineering Materials

The engineering materials are mainly classified as :

1. Metals and their alloys, such as iron, steel, copper, aluminium, etc.

2. Non-metals, such as glass, rubber, plastic, etc.
The metals may be further classified as :

(*a*) Ferrous metals, and (*b*) Non-ferrous metals.

The ***ferrous metals** are those which have the iron as their main constituent, such as cast iron, wrought iron and steel. The ***non-ferrous*** metals are those which have a metal other than iron as their main constituent, such as copper, aluminium, brass, tin, zinc, etc

Selection of Materials for Engineering Purposes

The selection of a proper material, for engineering purposes, is one of the most difficult problem for the designer. The best material is one which serve the desired objective at the minimum cost. The following factors should be considered while selecting the material :

1. Availability of the materials,
2. Suitability of the materials for the working conditions in service, and
3. The cost of the materials.

Mechanical Properties of Metals

The mechanical properties of the metals are those which are associated with the ability of the
material to resist mechanical forces and load. These mechanical properties of the metal include strength,
stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, resilience, creep and
hardness. We shall now discuss these properties as follows:

1. *Strength*. It is the ability of a material to resist the externally applied forces without breaking
or yielding. The internal resistance offered by a part to an externally applied force is called *stress.

2. *Stiffness*. It is the ability of a material to resist deformation under stress. The modulus of
elasticity is the measure of stiffness.

3. *Elasticity*. It is the property of a material to regain its original shape after deformation when
the external forces are removed. This property is desirable for materials used in tools and machines.
It may be noted that steel is more elastic than rubber.

4. *Plasticity*. It is property of a material which retains the deformation produced under load
permanently. This property of the material is necessary for forgings, in stamping images on coins and
in ornamental work.

5. *Ductility*. It is the property of a material enabling it to be drawn into wire with the application
of a tensile force. A ductile material must be both strong and plastic. The ductility is usually
measured by the terms, percentage elongation and percentage reduction in area. The ductile material
commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper,
aluminium, nickel, zinc, tin and lead.

6. *Brittleness*. It is the property of a material opposite to ductility. It is the property of breaking
of a material with little permanent distortion. Brittle materials when subjected to tensile loads, snap
off without giving any sensible elongation. Cast iron is a brittle material.

7. ***Malleability*****.** It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminium.

8. ***Toughness*****.** It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed upto the point of fracture. This property is desirable in parts subjected to shock and impact loads.

9. ***Machinability*****.** It is the property of a material which refers to a relative case with which a material can be cut. The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials or thrust required to remove the material at some given rate or the energy required to remove a unit volume of the material. It may be noted that brass can be easily machined than steel.

10. ***Resilience*****.** It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within elastic limit. This property is essential for spring materials.

11. ***Creep*****.** When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called ***creep*****.** This property is considered in designing internal combustion engines, boilers and turbines.

Completely Reversed or Cyclic Stresses

Consider a rotating beam of circular cross-section and carrying a load *W*, This load induces stresses in the beam which are cyclic in nature. *A* little consideration will show that the upper fibres of the beam (*i.e.* at point *A*) are under compressive stress and the lower fibres (*i.e.* at point *B*) are under tensile stress. After half a revolution, the point *B* occupies the position of point *A* and the point *A* occupies the position of point *B*. Thus the point *B* is now under compressive stress and the point *A* under tensile stress. The speed of variation of these stresses depends upon the speed of the beam.

From above we see that for each revolution of the beam, the stresses are reversed from compressive to tensile. The stresses which vary from one value of compressive to the same value of tensile or ***vice versa,*** are known as ***completely reversed*** or ***cyclic stresses.***

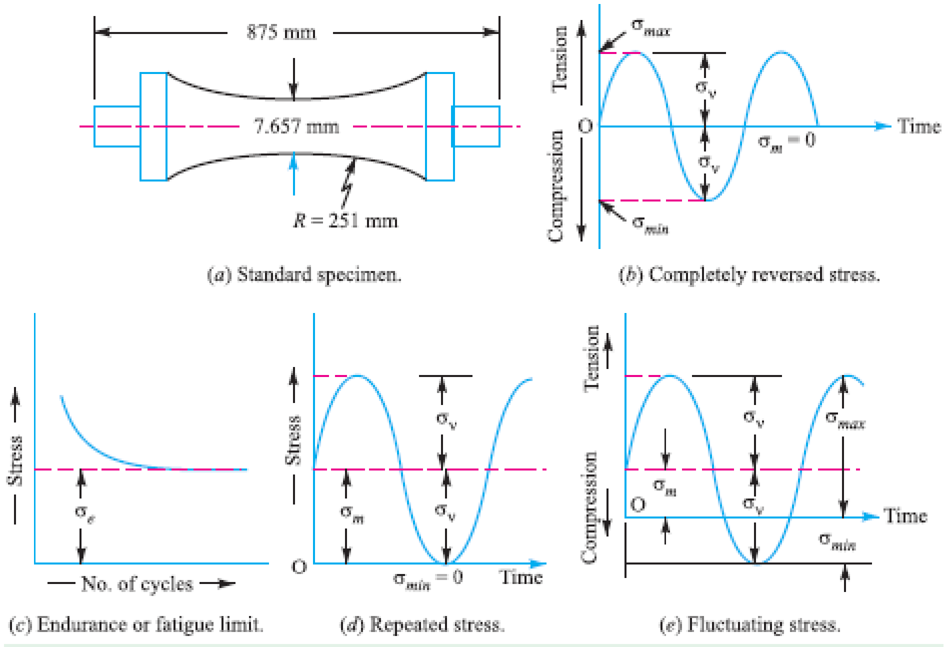
Notes: 1. The stresses which vary from a minimum value to a maximum value of the same nature, (*i.e*. tensile or compressive) are called ***fluctuating stresses.***

2. The stresses which vary from zero to a certain maximum value are called ***repeated stresses.***

3. The stresses which vary from a minimum value to a maximum value of the opposite nature (*i.e*. from a certain minimum compressive to a certain maximum tensile or from a minimum tensile to a maximum compressive) are called ***alternating stresses.***

Fatigue and Endurance Limit

It has been found experimentally that when a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue.** The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The failure may occur even without any prior indication. The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.



(a) Standard specimen. (b) Completely reversed stress.

(c) Endurance or fatigue limit. (d) Repeated stress. (e) Fluctuating stress.

Effect of Loading on Endurance Limit—Load Factor

The endurance limit (σe) of a material as determined by the rotating beam method is for reversed bending load. There are many machine members which are subjected to loads other than reversed bending loads. Thus the endurance limit will also be different for different types of loading. The endurance limit depending upon the type of loading may be modified as discussed below:

Let *Kb* = Load correction factor for the reversed or rotating bending load. Its value is usually taken as unity.

Ka = Load correction factor for the reversed axial load. Its value may be taken as 0.8.

Ks = Load correction factor for the reversed torsional or shear load. Its value may be taken as 0.55 for ductile materials and 0.8 for brittle materials.

4 Endurance limit for reversed bending load, $\sigma eb = \sigma e.Kb = \sigma e$ ...(QKb = 1) Endurance limit for reversed axial load, $\sigma ea = \sigma e.Ka$

and endurance limit for reversed torsional or shear load, $\tau e = \sigma e.Ks$

Effect of Surface Finish on Endurance Limit—Surface Finish Factor

When a machine member is subjected to variable loads, the endurance limit of the material for
that member depends upon the surface conditions. Fig. 6.3 shows the values of surface finish factor
for the various surface conditions and ultimate tensile strength.

Important Terms used in Limit System

The following terms used in limit system
(or interchangeable system) are important from
the subject point of view:

1. *Nominal size*. It is the size of a part
specified in the drawing as a matter of convenience.

2. *Basic size*. It is the size of a part to which
all limits of variation (*i.e*. tolerances) are applied
to arrive at final dimensioning of the mating parts.
The nominal or basic size of a part is often the
same.

3. *Actual size*. It is the actual measured
dimension of the part. The difference between the basic size and the actual size should not exceed a
certain limit, otherwise it will interfere with the interchangeability of the mating parts.

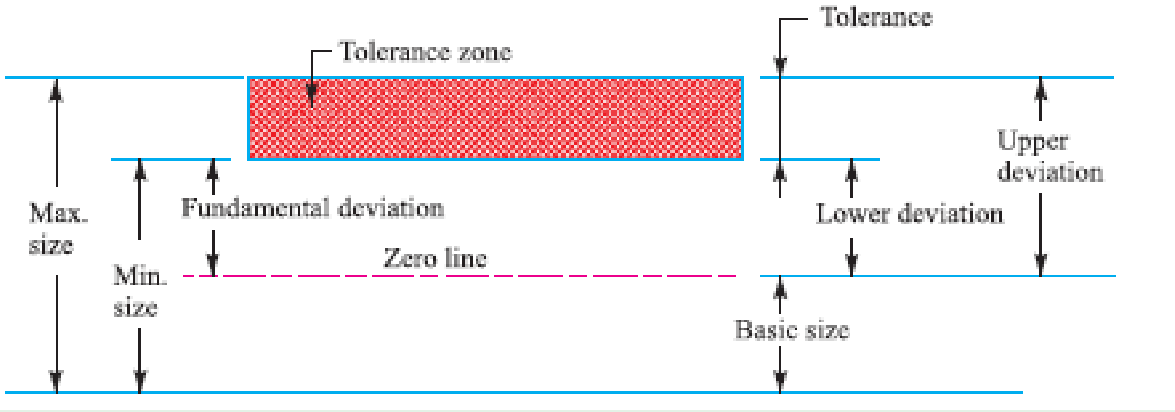
4. *Limits of sizes*. There are two extreme permissible sizes for a dimension of the part as
shown in Fig. 3.1. The largest permissible size for a dimension of the part is called ***upper*** or ***high*** or
maximum limit, whereas the smallest size of the part is known as ***lower*** or ***minimum limit.***

5. *Allowance*. It is the difference between the basic dimensions of the mating parts. The
allowance may be ***positive*** or ***negative***. When the shaft size is less than the hole size, then the allowance
is ***positive*** and when the shaft size is greater than the hole size, then the allowance is ***negative*.**

6. *Tolerance*. It is the difference between the upper limit and lower limit of a dimension. In

other words, it is the maximum permissible variation in a dimension.
The tolerance may be ***unilateral***
or ***bilateral*.** When all the tolerance is allowed on one side of the nominal size, *e*.*g*. 0.000
20– 0.004 + , then it
is said to be ***unilateral system of tolerance*.** The unilateral system is mostly used in industries as it
permits changing the tolerance value while still retaining the same allowance or type of fit.
When the tolerance is allowed on both sides of the nominal size, *e*.*g*. 0.002
20– 0.002 + , then it is said to
be ***bilateral system of tolerance*.** In this case + 0.002 is the upper limit and – 0.002 is the lower limit.
The method of assigning unilateral and bilateral tolerance is shown in Fig. 3.2 (*a*) and
(*b*) respectively.
7. *Tolerance zone*. It is the zone between the maximum and minimum limit size, as shown in
Fig. 3.3.
Fig. 3.3. Tolerance zone.
8. *Zero line*. It is a straight line corresponding to the basic size. The deviations are measured
from this line. The positive and negative deviations are shown above and below the zero line
respectively.
9. *Upper deviation*. It is the algebraic difference between the maximum size and the basic size.
The upper deviation of a hole is represented by a symbol *ES* (Ecart Superior) and of a shaft, it is
represented by *es*.
10. *Lower deviation*. It is the algebraic difference between the minimum size and the basic size.
The lower deviation of a hole is represented by a symbol *EI* (Ecart Inferior) and of a shaft, it is
represented by *ei*.
11. *Actual deviation*. It is the algebraic difference between an actual size and the corresponding
basic size.

12. *Mean deviation.* It is the arithmetical mean between the upper and lower deviations.
13. *Fundamental deviation.* It is one of the two deviations which is conventionally chosen to
define the position of the tolerance zone in relation to zero line, as shown in fig



Fits

The degree of tightness or looseness between the two mating parts is known as a *fit* of the parts.
The nature of fit is characterised by the presence and size of clearance and interference.
The ***clearance*** is the amount by which the actual size of the shaft is less than the actual size of
the mating hole in an assembly as shown in Fig. 3.5 (*a*). In other words, the clearance is the difference
between the sizes of the hole and the shaft before assembly. The difference must be ***positive*.**
Types of fits.
The ***interference*** is the amount by which the actual size of a shaft is larger than the actual
finished size of the mating hole in an assembly as shown in Fig. 3.5 (*b*). In other words, the interference
is the arithmetical difference between the sizes of the hole and the shaft, before assembly. The difference
must be ***negative*.**

Types of Fits

According to Indian standards, the fits are classified into the following three groups :

1. *Clearance fit.* In this type of fit, the size limits for mating parts are so selected that clearance
between them always occur, as shown in Fig. 3.5 (*a*). It may be noted that in a clearance fit, the
tolerance zone of the hole is entirely above the tolerance zone of the shaft.
In a clearance fit, the difference between the minimum size of the hole and the maximum size of
the shaft is known as ***minimum clearance*** whereas the difference between the maximum size of the
hole and minimum size of the shaft is called ***maximum clearance***

2. *Interference fit.* In this type of fit, the size limits for the mating parts are so selected that
interference between them always occur, as shown in Fig. 3.5 (*b*). It may be noted that in an interference
fit, the tolerance zone of the hole is entirely below the tolerance zone of the shaft.
In an interference fit, the difference between the maximum size of the hole and the minimum
size of the shaft is known as ***minimum interference,*** whereas the difference between the minimum
size of the hole and the maximum size of the shaft is called ***maximum interference,*** as shown in Fig.
3.5 (*b*).
The interference fits may be shrink fit, heavy drive fit and light drive fit.
3. *Transition fit.* In this type of fit, the size limits for the mating parts are so selected that either
a clearance or interference may occur depending upon the actual size of the mating parts, as shown in
Fig. 3.5 (*c*). It may be noted that in a transition fit, the tolerance zones of hole and shaft overlap.
The transition fits may be force fit, tight fit and push fit.

Effect of Surface Finish on Endurance Limit—Surface Finish Factor

When a machine member is subjected to variable loads, the endurance limit of the material for
that member depends upon the surface conditions. Fig. 6.3 shows the values of surface finish factor

for the various surface conditions and ultimate tensile strength.
Surface finish factor for various surface conditions.
When the surface finish factor is known, then the endurance limit for the material of the machine member may be obtained by multiplying the endurance limit and the surface finish factor. We see that for a mirror polished material, the surface finish factor is unity. In other words, the endurance limit for mirror polished material is maximum and it goes on reducing due to surface condition.

Let *Ksur* = Surface finish factor.

4 Endurance limit,

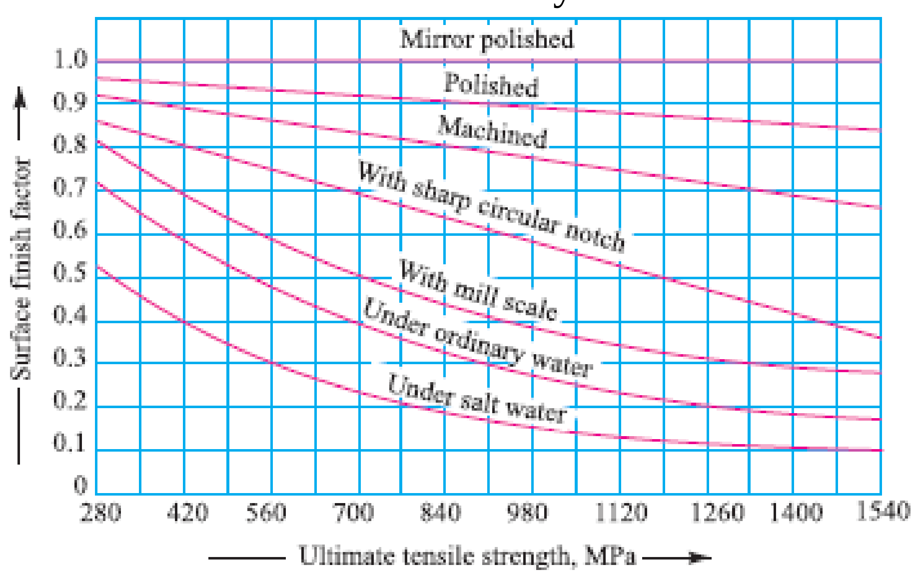
ʃ*e*1 = ʃ*eb*.*Ksur*= ʃ*e*.*Kb*.*Ksur* = ʃ*e*.*Ksur* ...(Q *Kb* = 1)

...(For reversed bending load)

= ʃ*ea*.*Ksur* = ʃ*e*.*Ka*.*Ksur* ...(For reversed axial load)

= | *e*.*Ksur* = ʃ*e*.*Ks*.*Ksur* ...(For reversed torsional or shear load)

Note : The surface finish factor for non-ferrous metals may be taken as unity.



Rankine's Formula for Columns

We have already discussed that Euler's formula gives correct results only for very long columns.
Though this formula is applicable for columns, ranging from very long to short ones, yet it does not give reliable results. Prof. Rankine, after a number of experiments, gave the following empirical formula for columns

$$\frac{1}{W_{cr}} = \frac{1}{W_C} + \frac{1}{W_E}$$

where W_{cr} = Crippling load by Rankine's formula,

W_C = Ultimate crushing load for the column = $\sigma_c \times A$,

W_E = Crippling load, obtained by Euler's formula = $\dfrac{\pi^2 E I}{L^2}$

A little consideration will show, that the value of *W*C will remain constant irrespective of the fact whether the column is a long one or short one. Moreover, in the case of short columns, the value of *W*E will be very high, therefore the value of 1 / *W*E will be quite negligible as compared to 1 / *W*C.

It is thus obvious, that the Rankine's formula will give the value of its crippling load (*i.e. Wcr*) approximately equal to the ultimate crushing load (*i.e. W*C). In case of long columns, the value of *W*E will be very small, therefore the value of 1 / *W*E will be quite considerable as compared to 1 / *W*C. It is thus obvious, that the Rankine's formula will give the value of its crippling load (*i.e. Wcr*) approximately equal to the crippling load by Euler's formula (*i.e. W*E). Thus, we see that Rankine's formula gives a fairly correct result for all cases of columns, ranging from short to long columns.

From equation (*i*), we know that *L* = Equivalent length of the column, and

k = Least radius of gyration.

The following table gives the values of crushing stress and Rankine's constant for various materials.

From equation (i), we know that

$$\frac{1}{W_{cr}} = \frac{1}{W_C} + \frac{1}{W_E} = \frac{W_E + W_C}{W_C \times W_E}$$

$$\therefore \qquad W_{cr} = \frac{W_C \times W_E}{W_C + W_E} = \frac{W_C}{1 + \dfrac{W_C}{W_E}}$$

Now substituting the value of W_C and W_E in the above equation, we have

$$W_{cr} = \frac{\sigma_c \times A}{1 + \dfrac{\sigma_c \times A \times L^2}{\pi^2 E I}} = \frac{\sigma_c \times A}{1 + \dfrac{\sigma_c}{\pi^2 E} \times \dfrac{A L^2}{A k^2}} \qquad \ldots (\because I = A.k^2)$$

$$= \frac{\sigma_c \times A}{1 + a\left(\dfrac{L}{k}\right)^2} = \frac{\text{Crushing load}}{1 + a\left(\dfrac{L}{k}\right)^2}$$

where σ_c = Crushing stress or yield stress in compression,

A = Cross-sectional area of the column,

a = Rankine's constant $= \dfrac{\sigma_c}{\pi^2 E}$,

Johnson's Formulae for Columns

Prof. J.B. Johnson proposed the following two formula for short columns.

1. *Straight line formula.* According to straight line formula proposed by Johnson, the critical or crippling load is

$$W_{cr} = A\left[\sigma_y - \frac{2\,\sigma_y}{3\pi}\left(\frac{L}{k}\right)\sqrt{\frac{\sigma_y}{3C \times E}}\right] = A\left[\sigma_y - C_1\left(\frac{L}{k}\right)\right]$$

where A = Cross-sectional area of column,

σ_y = Yield point stress,

$$C_1 = \frac{2\,\sigma_y}{3\pi}\sqrt{\frac{\sigma_y}{3C.E}}$$

= A constant, whose value depends upon the type of material as well as the type of ends, and

$\frac{L}{k}$ = Slenderness ratio.

If the safe stress (W_{cr} / A) is plotted against slenderness ratio (L / k), it works out to be a straight line, so it is known as straight line formula.

2. *Parabolic formula.* Prof. Johnson after proposing the straight line formula found that the results obtained by this formula are very approximate. He then proposed another formula, according to which the critical or crippling load,

$$W_{cr} = A \times \sigma_y\left[1 - \frac{\sigma_y}{4\,C\,\pi^2\,E}\left(\frac{L}{k}\right)^2\right] \text{ with usual notations.}$$

If a curve of safe stress (W_{cr} / A) is plotted against (L / k), it works out to be a parabolic, so it is known as parabolic formula.

Fig. 16.4 shows the relationship of safe stress (W_{cr} / A) and the slenderness ratio (L / k) as given by Johnson's formula and Euler's formula for a column made of mild steel with both ends hinged (*i.e.* C = 1), having a yield strength, σ_y = 210 MPa. We see from the figure that point A (the point of tangency between the Johnson's straight line formula and Euler's formula) describes the use of two formulae. In other words, Johnson's straight line formula may be used when $L / k < 180$ and the Euler's formula is used when $L / k > 180$.

UNIT II
DESIGN OF CYLINDER, PISTON AND CONNECTING ROD

Principal Parts of an Engine

The principal parts of an I.C engine, as shown in Fig. 32.1 are as follows :

1. Cylinder and cylinder liner, 2. Piston, piston rings and piston pin or gudgeon pin, 3. Connecting rod with small and big end bearing, 4. Crank, crankshaft and crank pin, and 5. Valve gear mechanism.

The design of the above mentioned principal parts are discussed, in detail, in the following pages.

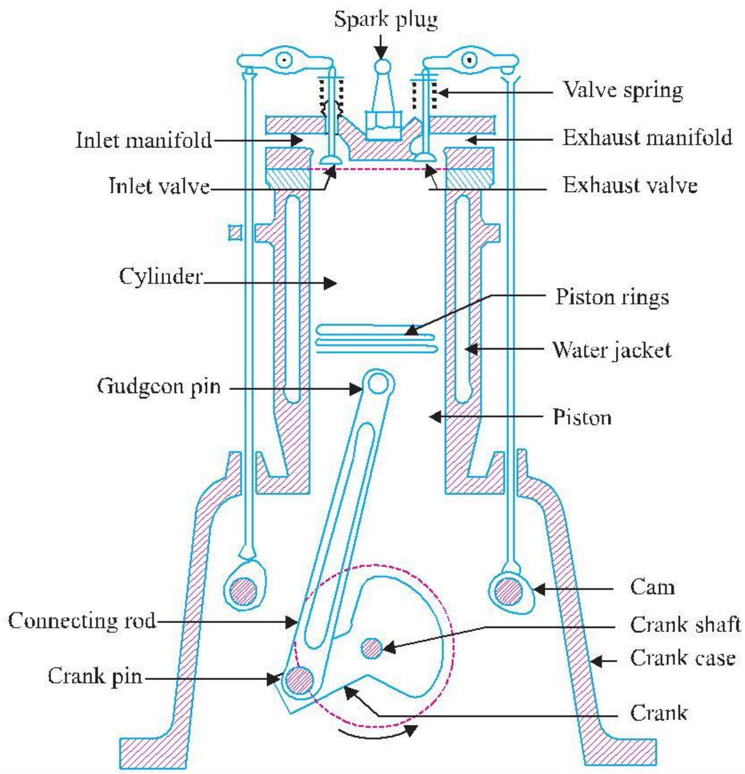
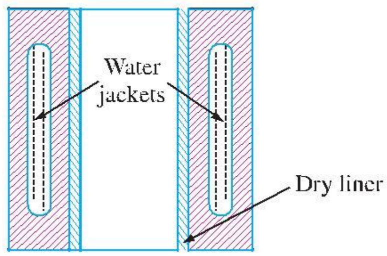


Fig. 32.1. Internal combustion engine parts.

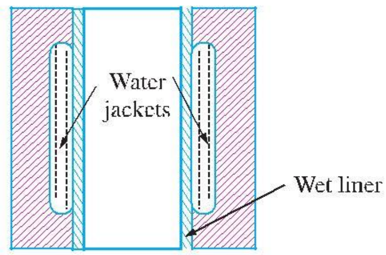
32.3 Cylinder and Cylinder Liner

The function of a cylinder is to retain the working fluid and to guide the piston. The cylinders are usually made of cast iron or cast steel. Since the cylinder has to withstand high temperature due to the combustion of fuel, therefore, some arrangement must be provided to cool the cylinder. The single cylinder engines (such as scooters and motorcycles) are generally air cooled. They are provided with fins around the cylinder. The multi-cylinder engines (such as of cars) are provided with water jackets around the cylinders to cool it. In smaller engines. the cylinder, water jacket and the frame are made as one piece, but for all the larger engines, these parts are manufactured separately. The cylinders are provided with cylinder liners so that in case of wear, they can be easily replaced. The cylinder liners are of the following two types :

1. Dry liner, and **2.** Wet liner.



(*a*) Dry liner.



(*b*) Wet liner.

Fig. 32.2. Dry and wet liner.

A cylinder liner which does not have any direct contact with the engine cooling water, is known as ***dry liner,*** as shown in Fig. 32.2 (*a*). A cylinder liner which have its outer surface in direct contact with the engine cooling water, is known as ***wet liner,*** as shown in Fig. 32.2 (*b*).

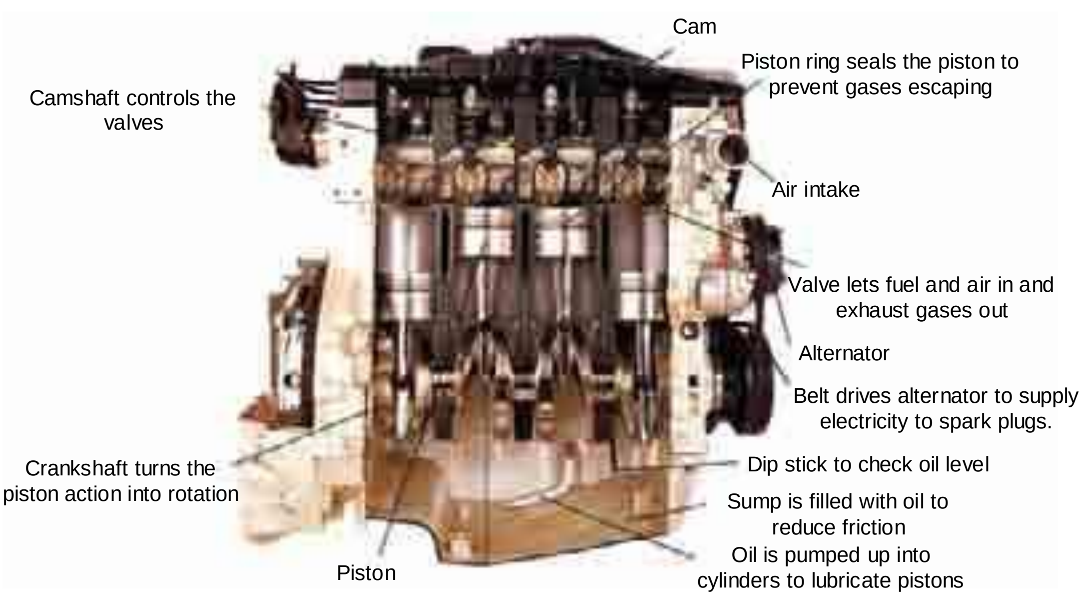
The cylinder liners are made from good quality close grained cast iron (*i.e.* pearlitic cast iron), nickel cast iron, nickel chromium cast iron. In some cases, nickel chromium cast steel with molybdenum may be used. The inner surface of the liner should be properly heat-treated in order to obtain a hard surface to reduce wear.

Design of a Cylinder

In designing a cylinder for an I. C. engine, it is required to determine the following values :

1. *Thickness of the cylinder wall.* The cylinder wall is subjected to gas pressure and the piston side thrust. The gas pressure produces the following two types of stresses :

(a) Longitudinal stress, and ***(b)*** Circumferential stress.



The above picture shows crankshaft, pistons and cylinder of a 4-stroke petrol engine.

Since these two stressess act at right angles to each other, therefore, the net stress in each direction is reduced.

The piston side thrust tends to bend the cylinder wall, but the stress in the wall due to side thrust is very small and hence it may be neglected.

Let D_0 = Outside diameter of the cylinder in mm,

D = Inside diameter of the cylinder in mm,

p = Maximum pressure inside the engine cylinder in N/mm^2,

t = Thickness of the cylinder wall in mm, and

$1/m$ = Poisson's ratio. It is usually taken as 0.25.

The apparent longitudinal stress is given by

$$\sigma_l = \frac{\text{Force}}{\text{Area}} = \frac{\frac{\pi}{4} \cdot D^2 \cdot p}{\frac{\pi}{4}\left[(D_0)^2 - D^2\right]} = \frac{D^2 \cdot p}{(D_0)^2 - D^2}$$

and the apparent circumferential stresss is given by

$$\sigma_c = \frac{\text{Force}}{\text{Area}} = \frac{D \cdot l \cdot p}{2t \cdot l} = \frac{D \cdot p}{2t}$$

... (where l is the length of the cylinder and area is the projected area)

$\therefore$ Net longitudinal stress $= \sigma_l - \dfrac{\sigma_c}{m}$

and net circumferential stress $= \sigma_c - \dfrac{\sigma_l}{m}$

The thickness of a cylinder wall (t) is usually obtained by using a thin cylindrical formula, *i.e.*,

$$t = \frac{p \cdot D}{2\,\sigma_c} + C$$

where p = Maximum pressure inside the cylinder in N/mm^2,

D = Inside diameter of the cylinder or cylinder bore in mm,

σ_c = Permissible circumferential or hoop stress for the cylinder material in MPa or N/mm^2. Its value may be taken from 35 MPa to 100 MPa depending upon the size and material of the cylinder.

C = Allowance for reboring.

The allowance for reboring (C) depending upon the cylinder bore (D) for I. C. engines is given in the following table :

Table 32.1. Allowance for reboring for I. C. engine cylinders.

D (mm)	75	100	150	200	250	300	350	400	450	500
C (mm)	1.5	2.4	4.0	6.3	8.0	9.5	11.0	12.5	12.5	12.5

The thickness of the cylinder wall usually varies from 4.5 mm to 25 mm or more depending upon the size of the cylinder. The thickness of the cylinder wall (t) may also be obtained from the following empirical relation, *i.e.*

$$t = 0.045\,D + 1.6 \text{ mm}$$

The other empirical relations are as follows :

Thickness of the dry liner

$$= 0.03\,D \text{ to } 0.035\,D$$

Thickness of the water jacket wall

= 0.032 D + 1.6 mm or t / 3 m for bigger cylinders and 3t /4 for smaller cylinders

Water space between the outer cylinder wall and inner jacket wall

= 10 mm for a 75 mm cylinder to 75 mm for a 750 mm cylinder or 0.08 D + 6.5 mm

2. ***Bore and length of the cylinder.*** The bore (*i.e.* inner diameter) and length of the cylinder may be determined as discussed below :

Let p_m = Indicated mean effective pressure in N/mm^2,

D = Cylinder bore in mm,

A = Cross-sectional area of the cylinder in mm^2,

$= \pi D^2/4$

l = Length of stroke in metres,

N = Speed of the engine in r.p.m., and

n = Number of working strokes per min

= N, for two stroke engine

= N/2, for four stroke engine.

We know that the power produced inside the engine cylinder, *i.e.* indicated power,

$$I.P. = \frac{p_m \cdot l \cdot A \cdot n}{60} \text{ watts}$$

From this expression, the bore (D) and length of stroke (l) is determined. The length of stroke is generally taken as 1.25 D to 2D.

Since there is a clearance on both sides of the cylinder, therefore length of the cylinder is taken as 15 percent greater than the length of stroke. In other words,

Length of the cylinder, L = 1.15 × Length of stroke = 1.15 l

Notes : (*a*) If the power developed at the crankshaft, *i.e.* brake power (*B. P.*) and the mechanical efficiency (η_m) of the engine is known, then

$$I.P. = \frac{B.P.}{\eta_m}$$

(*b*) The maximum gas pressure (p) may be taken as 9 to 10 times the mean effective pressure (p_m).

3. ***Cylinder flange and studs.*** The cylinders are cast integral with the upper half of the crank-case or they are attached to the crankcase by means of a flange with studs or bolts and nuts. The cylinder flange is integral with the cylinder and should be made thicker than the cylinder wall. The flange thickness should be taken as 1.2 t to 1.4 t, where t is the thickness of cylinder wall.

The diameter of the studs or bolts may be obtained by equating the gas load due to the maximum pressure in the cylinder to the resisting force offered by all the studs or bolts. Mathematically,

$$\frac{\pi}{4} \cdot D^2 \cdot p = n_s \cdot \frac{\pi}{4} (d_c)^2 \sigma_T$$

where D = Cylinder bore in mm,

p = Maximum pressure in N/mm^2,

n_s = Number of studs. It may be taken as 0.01 D + 4 to 0.02 D + 4

d_c = Core or minor diameter, *i.e.* diameter at the root of the thread in mm,

σ_t = Allowable tensile stress for the material of studs or bolts in MPa or N/mm^2. It may be taken as 35 to 70 MPa.

The nominal or major diameter of the stud or bolt (d) usually lies between 0.75 t_f to t_f, where t_f is the thickness of flange. In no case, a stud or bolt less than 16 mm diameter should be used.

The distance of the flange from the centre of the hole for the stud or bolt should not be less than d + 6 mm and not more than 1.5 d, where d is the nominal diameter of the stud or bolt.

In order to make a leak proof joint, the pitch of the studs or bolts should lie between $19\sqrt{d}$ to $28.5\sqrt{d}$, where d is in mm.

4. *Cylinder head.* Usually, a separate cylinder head or cover is provided with most of the engines. It is, usually, made of box type section of considerable depth to accommodate ports for air and gas passages, inlet valve, exhaust valve and spark plug (in case of petrol engines) or atomiser at the centre of the cover (in case of diesel engines).

The cylinder head may be approximately taken as a flat circular plate whose thickness (t_h) may be determined from the following relation :

$$t_h = D\sqrt{\frac{C \cdot p}{\sigma_c}}$$

where D = Cylinder bore in mm,

p = Maximum pressure inside the cylinder in N/mm^2,

σ_c = Allowable circumferential stress in MPa or N/mm^2. It may be taken as 30 to 50 MPa, and

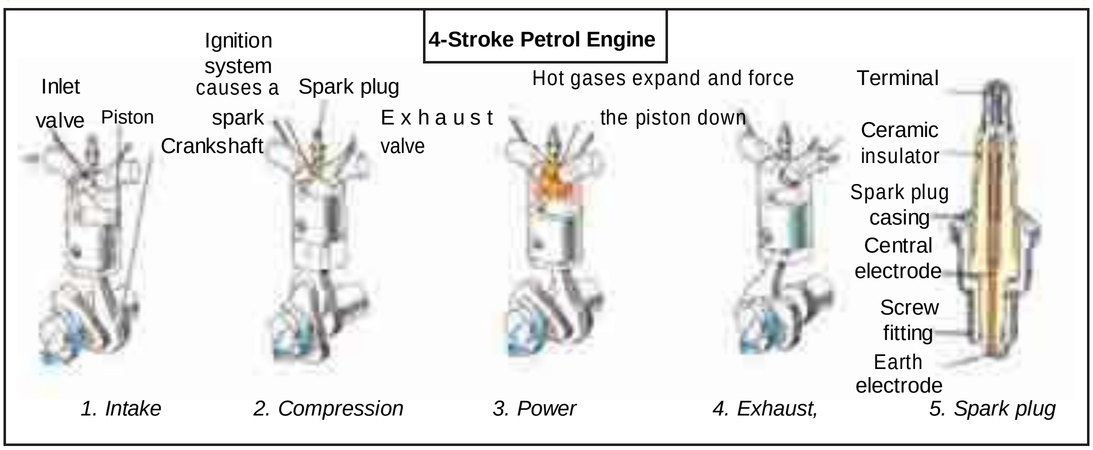
C = Constant whose value is taken as 0.1.

The studs or bolts are screwed up tightly alongwith a metal gasket or asbestos packing to provide a leak proof joint between the cylinder and cylinder head. The tightness of the joint also depends upon the pitch of the bolts or studs, which should lie between $19\sqrt{d}$ to $28.5\sqrt{d}$. The pitch circle diameter (D_p) is usually taken as $D + 3d$. The studs or bolts are designed in the same way as discussed above.

Example 32.1. *A four stroke diesel engine has the following specifications :*

Brake power = 5 kW ; Speed = 1200 r.p.m. ; Indicated mean effective pressure = 0.35 N / mm^2 ; Mechanical efficiency = 80 %.

Determine : 1. bore and length of the cylinder ; 2. thickness of the cylinder head ; and 3. size of studs for the cylinder head.



Solution. Given: $B.P. = 5\text{kW} = 5000$ W ; $N = 1200$ r.p.m. or $n = N/2 = 600$; $p_m = 0.35$ N/mm^2; $\eta_m = 80\% = 0.8$

1. ***Bore and length of cylinder***

Let D = Bore of the cylinder in mm,

A = Cross-sectional area of the cylinder $= \frac{\pi}{4} \cdot D^2$ mm^2

l = Length of the stroke in m.

$= 1.5\,D$ mm $= 1.5\,D / 1000$ m ....(Assume)

We know that the indicated power,

$$I.P = B.P. / \eta_m = 5000 / 0.8 = 6250 \text{ W}$$

We also know that the indicated power ($I.P.$),

$$6250 = \frac{p_m \cdot l \cdot A \cdot n}{60} = \frac{0.35 \cdot 1.5\,D \cdot \pi D^2 \cdot 600}{60 \cdot 1000 \cdot 4} = 4.12 \times 10^{-3}\,D^3$$

...($\because$ For four stroke engine, $n = N/2$)

$\therefore$ $D^3 = 6250 / 4.12 \times 10^{-3} = 1517 \times 10^3$ or $D = 115$ mm **Ans.**

and $l = 1.5\,D = 1.5 \times 115 = 172.5$ mm

Taking a clearance on both sides of the cylinder equal to 15% of the stroke, therefore length of the cylinder,

$$L = 1.15\,l = 1.15 \times 172.5 = 198 \text{ say } 200 \text{ mm } \textbf{Ans.}$$

2. ***Thickness of the cylinder head***

Since the maximum pressure (p) in the engine cylinder is taken as 9 to 10 times the mean effective pressure (p_m), therefore let us take

$$p = 9\,p_m = 9 \times 0.35 = 3.15 \text{ N/mm}^2$$

We know that thickness of the cyclinder head,

$$t_h = D\sqrt{\frac{C \cdot p}{\sigma_t}} = 115\sqrt{\frac{0.1 \cdot 3.15}{42}} = 9.96 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

...(Taking $C = 0.1$ and $\sigma_t = 42$ MPa $= 42$ N/mm^2)

3. ***Size of studs for the cylinder head***

Let d = Nominal diameter of the stud in mm,

d_c = Core diameter of the stud in mm. It is usually taken as $0.84\,d$.

σ_t = Tensile stress for the material of the stud which is usually nickel steel.

n_s = Number of studs.

We know that the force acting on the cylinder head (or on the studs)

$$= \frac{\pi}{4} \cdot D^2 \cdot p = \frac{\pi}{4}(115)^2\, 3.15 = 32\ 702 \text{ N} \quad ...(i)$$

The number of studs (n_s) are usually taken between $0.01\,D + 4$ (*i.e.* $0.01 \times 115 + 4 = 5.15$) and $0.02\,D + 4$ (*i.e.* $0.02 \times 115 + 4 = 6.3$). Let us take $n_s = 6$.

We know that resisting force offered by all the studs

$$= n_s \cdot \frac{\pi}{4}(d_c)^2\,\sigma_t = 6 \cdot \frac{\pi}{4}(0.84d)^2\, 65 = 216\,d^2 \text{ N} \quad ...(ii)$$

...(Taking $\sigma_t = 65$ MPa $= 65$ N/mm^2)

From equations ***(i)*** and ***(ii)***,

$$d^2 = 32\ 702 / 216 = 151 \text{ or } d = 12.3 \text{ say } 14 \text{ mm}$$

The pitch circle diameter of the studs (D_p) is taken $D + 3d$.

$\therefore \quad D_p = 115 + 3 \times 14 = 157$ mm

We know that pitch of the studs

$$= \frac{\pi \cdot D_p}{n_s} = \frac{\pi \cdot 157}{6} = 82.2 \text{ mm}$$

We know that for a leak-proof joint, the pitch of the studs should lie between $19\sqrt{d}$ to $28.5\sqrt{d}$, where d is the nominal diameter of the stud.

$\therefore$ Minimum pitch of the studs

$$= 19\sqrt{d} = 19\sqrt{14} = 71.1 \text{ mm and}$$

maximum pitch of the studs

$$= 28.5\sqrt{d} = 28.5\sqrt{14} = 106.6 \text{mm}$$

Since the pitch of the studs obtained above (*i.e.* 82.2 mm) lies within 71.1 mm and 106.6 mm, therefore, size of the stud (d) calculated above is satisfactory.

$d = 14$ mm **Ans.**

32.5 Piston

The piston is a disc which reciprocates within a cylinder. It is either moved by the fluid or it moves the fluid which enters the cylinder. The main function of the piston of an internal combustion engine is to receive the impulse from the expanding gas and to transmit the energy to the crankshaft through the connecting rod. The piston must also disperse a large amount of heat from the combustion chamber to the cylinder walls.

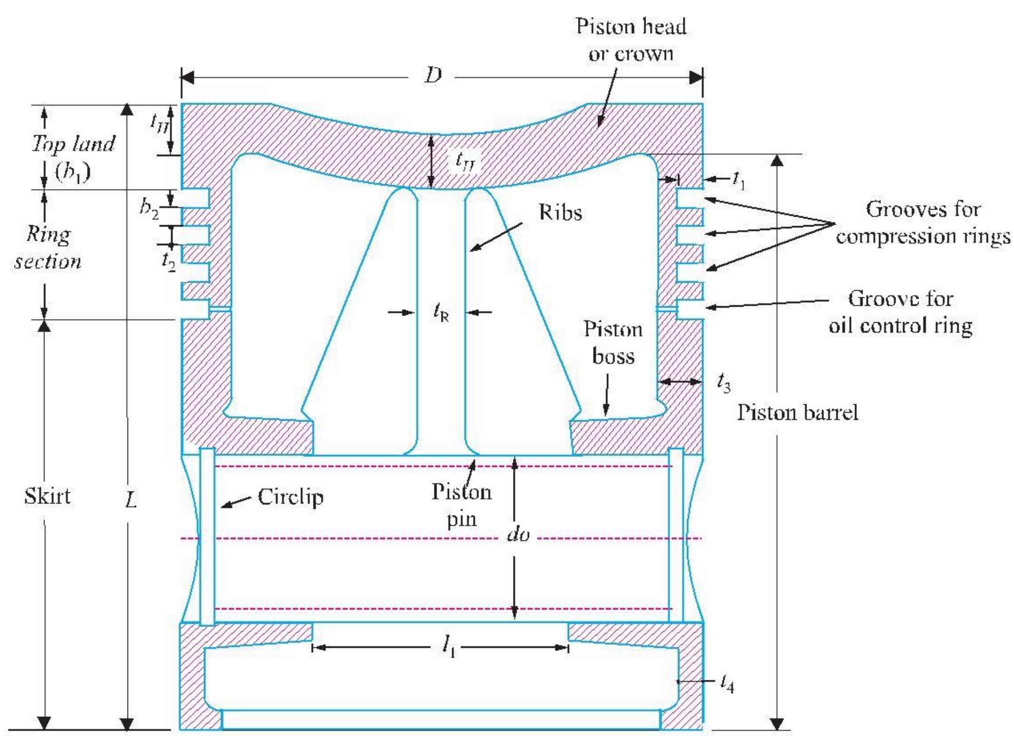


Fig. 32.3. Piston for I.C. engines (Trunk type).

The piston of internal combustion engines are usually of trunk type as shown in Fig. 32.3. Such pistons are open at one end and consists of the following parts :

1. *Head or crown.* The piston head or crown may be flat, convex or concave depending upon the design of combustion chamber. It withstands the pressure of gas in the cylinder.

2. *Piston rings.* The piston rings are used to seal the cyliner in order to prevent leakage of the gas past the piston.

3. *Skirt.* The skirt acts as a bearing for the side thrust of the connecting rod on the walls of cylinder.

4. *Piston pin.* It is also called ***gudgeon pin*** or ***wrist pin.*** It is used to connect the piston to the connecting rod.

32.6 Design Considerations for a Piston

In designing a piston for I.C. engine, the following points should be taken into consideration :

1. It should have enormous strength to withstand the high gas pressure and inertia forces.

2. It should have minimum mass to minimise the inertia forces.

3. It should form an effective gas and oil sealing of the cylinder.

4. It should provide sufficient bearing area to prevent undue wear.

5. It should disprese the heat of combustion quickly to the cylinder walls.

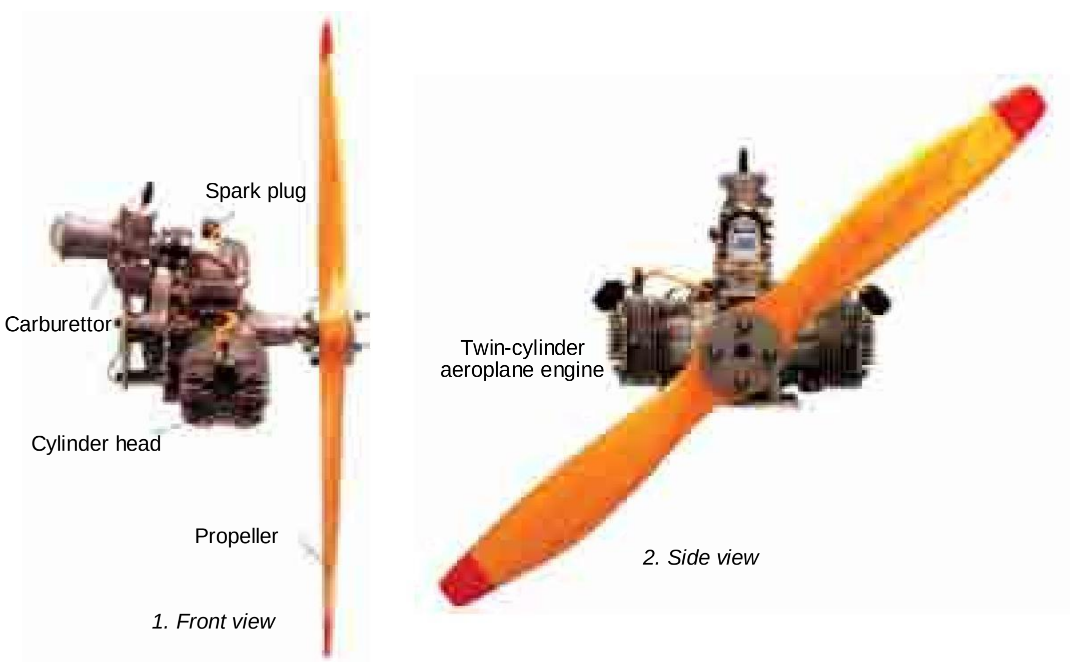
6. It should have high speed reciprocation without noise.

7. It should be of sufficient rigid construction to withstand thermal and mechanical distortion.

8. It should have sufficient support for the piston pin.

32.7 Material for Pistons

The most commonly used materials for pistons of I.C. engines are cast iron, cast aluminium, forged aluminium, cast steel and forged steel. The cast iron pistons are used for moderately rated



Twin cylinder airplane engine of 1930s.

engines with piston speeds below 6 m / s and aluminium alloy pistons are used for highly rated engines running at higher piston sppeds. It may be noted that

1. Since the *coefficient of thermal expansion for aluminium is about 2.5 times that of cast iron, therefore, a greater clearance must be provided between the piston and the cylinder wall (than with cast iron piston) in order to prevent siezing of the piston when engine runs continuously under heavy loads. But if excessive clearance is allowed, then the piston will develop ***'piston slap'*** while it is cold and this tendency increases with wear. The less clearance between the piston and the cylinder wall will lead to siezing of piston.

2. Since the aluminium alloys used for pistons have high **heat conductivity (nearly four times that of cast iron), therefore, these pistons ensure high rate of heat transfer and thus keeps down the maximum temperature difference between the centre and edges of the piston head or crown.

Notes: ***(a)*** For a cast iron piston, the temperature at the centre of the piston head (T_C) is about 425°C to 450°C under full load conditions and the temperature at the edges of the piston head (T_E) is about 200°C to 225°C.

(b) For aluminium alloy pistons, T_C is about 260°C to 290°C and T_E is about 185°C to 215°C.

3. Since the aluminium alloys are about ***three times lighter than cast iron, therfore, its mechanical strength is good at low temperatures, but they lose their strength (about 50%) at temperatures above 325°C. Sometimes, the pistons of aluminium alloys are coated with aluminium oxide by an electrical method.

32.8 Piston Head or Crown

The piston head or crown is designed keeping in view the following two main considerations, *i.e.*

1. It should have adequate strength to withstand the straining action due to pressure of explosion inside the engine cylinder, and

2. It should dissipate the heat of combustion to the cylinder walls as quickly as possible.

On the basis of first consideration of straining action, the thickness of the piston head is determined by treating it as a flat circular plate of uniform thickness, fixed at the outer edges and subjected to a uniformly distributed load due to the gas pressure over the entire cross-section.

The thickness of the piston head (t_H), according to Grashoff's formula is given by

$$t_H = \sqrt{\frac{3\,p.D^2}{16\sigma_t}} \text{ (in mm)} \qquad \ldots(i)$$

where p = Maximum gas pressure or explosion pressure in N/mm^2,

D = Cylinder bore or outside diameter of the piston in mm, and

σ_t = Permissible bending (tensile) stress for the material of the piston in MPa or N/mm^2. It may be taken as 35 to 40 MPa for grey cast iron, 50 to 90 MPa for nickel cast iron and aluminium alloy and 60 to 100 MPa for forged steel.

On the basis of second consideration of heat transfer, the thickness of the piston head should be such that the heat absorbed by the piston due combustion of fuel is quickly transferred to the cylinder walls. Treating the piston head as a flat ciucular plate, its thickness is given by

$$t_H = \frac{H}{12.56k\,(T_C - T_E)} \text{ (in mm)} \qquad \ldots(ii)$$

* The coefficient of thermal expansion for aluminium is 0.24×10^{-6} m / °C and for cast iron it is 0.1×10^{-6} m / °C.

** The heat conductivity for aluminium is 174.75 W/m/°C and for cast iron it is 46.6 W/m /°C.

*** The density of aluminium is 2700 kg / m^3 and for cast iron it is 7200 kg / m^3.

where H = Heat flowing through the piston head in kJ/s or watts,

k = Heat conductivity factor in W/m/°C. Its value is 46.6 W/m/°C for grey cast iron, 51.25 W/m/°C for steel and 174.75 W/m/°C for aluminium alloys.

T_C = Temperture at the centre of the piston head in °C, and

T_E = Temperature at the edges of the piston head in °C.

The temperature difference ($T_C - T_E$) may be taken as 220°C for cast iron and 75°C for aluminium.

The heat flowing through the positon head (H) may be determined by the following expression, *i.e.,*

$$H = C \times HCV \times m \times B.P. \text{ (in kW)}$$

where C = Constant representing that portion of the heat supplied to the engine which is absorbed by the piston. Its value is usually taken as 0.05.

HCV = Higher calorific value of the fuel in kJ/kg. It may be taken as 45×10^3 kJ/kg for diesel and 47×10^3 kJ/ kg for petrol,

m = Mass of the fuel used in kg per brake power per second, and

$B.P.$ = Brake power of the engine per cylinder

Notes : 1. The thickness of the piston head (t_H) is calculated by using equations ***(i)*** and ***(ii)*** and larger of the two values obtained should be adopted.

2. When t_H is 6 mm or less, then no ribs are required to strengthen the piston head against gas loads. But when t_H is greater then 6 mm, then a suitable number of ribs at the centre line of the boss extending around the skirt should be provided to distribute the side thrust from the connecting rod and thus to prevent distortion of the skirt. The thickness of the ribs may be takes as $t_H / 3$ to $t_H / 2$.

3. For engines having length of stroke to cylinder bore (L / D) ratio upto 1.5, a cup is provided in the top of the piston head with a radius equal to 0.7 D. This is done to provide a space for combustion chamber.

32.9 Piston Rings

The piston rings are used to impart the necessary radial pressure to maintain the seal between the piston and the cylinder bore. These are usually made of grey cast iron or alloy cast iron because of their good wearing properties and also they retain spring characteristics even at high temperatures. The piston rings are of the following two types :

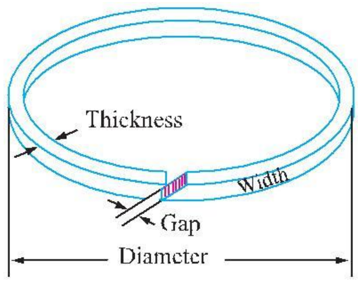
1. Compression rings or pressure rings, and

2. Oil control rings or oil scraper.

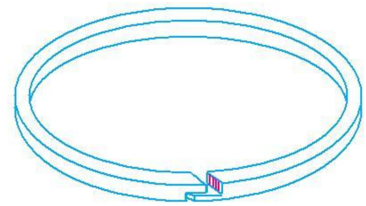
The ***compression rings*** *or* ***pressure rings*** are inserted in the grooves at the top portion of the piston and may be three to seven in number. These rings also transfer heat from the piston to the cylinder liner and absorb some part of the piston fluctuation due to the side thrust.

The ***oil control rings*** or ***oil scrapers*** are provided below the compression rings. These rings provide proper lubrication to the liner by allowing sufficient oil to move up during upward stroke and at the same time scraps the lubricating oil from the surface of the liner in order to minimise the flow of the oil to the combustion chamber.

The compression rings are usually made of rectangular cross-section and the diameter of the ring is slightly larger than the cylinder bore. A part of the ring is cut- off in order to permit it to go into the cylinder against the liner wall. The diagonal cut or step cut ends, as shown in Fig. 32.4 (*a*) and (*b*) respectively, may be used. The gap between the ends should be sufficiently large when the ring is put cold so that even at the highest temperature, the ends do not touch each other when the ring expands, otherwise there might be buckling of the ring.



(*a*) Diagonal cut.



(*b*) Step cut.

Fig. 32.4. Piston rings.

The radial thickness (t_1) of the ring may be obtained by considering the radial pressure between the cylinder wall and the ring. From bending stress consideration in the ring, the radial thickness is given by

$$t_1 = D\sqrt{\frac{3\,p_w}{\sigma_t}}$$

where D = Cylinder bore in mm,

p_w = Pressure of gas on the cylinder wall in N/mm^2. Its value is limited from 0.025 N/mm^2 to 0.042 N/mm^2, and

σ_t = Allowable bending (tensile) stress in MPa. Its value may be taken from 85 MPa to 110 MPa for cast iron rings.

The axial thickness (t_2) of the rings may be taken as 0.7 t_1 to t_1.

The minimum axial thickness (t_2) may also be obtained from the following empirical relation:

$$t_2 = \frac{D}{10 n_R}$$

where n_R = Number of rings.

The width of the top land (*i.e.* the distance from the top of the piston to the first ring groove) is made larger than other ring lands to protect the top ring from high temperature conditions existing at the top of the piston,

∴ Width of top land,

$$b_1 = t_H \text{ to } 1.2\, t_H$$

The width of other ring lands (*i.e.* the distance between the ring grooves) in the piston may be made equal to or slightly less than the axial thickness of the ring (t_2).

∴ Width of other ring lands,

$$b_2 = 0.75\, t_2 \text{ to } t_2$$

The depth of the ring grooves should be more than the depth of the ring so that the ring does not take any piston side thrust.

The gap between the free ends of the ring is given by 3.5 t_1 to 4 t_1. The gap, when the ring is in the cylinder, should be 0.002 D to 0.004 D.

32.10 Piston Barrel

It is a cylindrical portion of the piston. The maximum thickness (t_3) of the piston barrel may be obtained from the following empirical relation :

$$t_3 = 0.03\, D + b + 4.5 \text{ mm}$$

where b = Radial depth of piston ring groove which is taken as 0.4 mm larger than the radial thickness of the piston ring (t_1)

$= t_1 + 0.4$ mm

Thus, the above relation may be written as

$$t_3 = 0.03\,D + t_1 + 4.9 \text{ mm}$$

The piston wall thickness (t_4) towards the open end is decreased and should be taken as 0.25 t_3 to 0.35 t_3.

32.11 Piston Skirt

The portion of the piston below the ring section is known as ***piston skirt.*** In acts as a bearing for the side thrust of the connecting rod. The length of the piston skirt should be such that the bearing pressure on the piston barrel due to the side thrust does not exceed 0.25 N/mm^2 of the projected area for low speed engines and 0.5 N/mm^2 for high speed engines. It may be noted that the maximum thrust will be during the expansion stroke. The side thrust (R) on the cylinder liner is usually taken as 1/10 of the maximum gas load on the piston.

We know that maximum gas load on the piston,

$$P = p \cdot \frac{\pi D^2}{4}$$

∴ Maximum side thrust on the cylinder,

$$R = P/10 = 0.1\,p \cdot \frac{\pi D^2}{4} \qquad \ldots(i)$$

where p = Maximum gas pressure in N/mm^2, and

D = Cylinder bore in mm.

The side thrust (R) is also given by

R = Bearing pressure × Projected bearing area of the piston skirt

$= p_b \times D \times l$

where l = Length of the piston skirt in mm. ...(*ii*)

From equations **(*i*)** and **(*ii*)**, the length of the piston skirt (l) is determined. In actual practice, the length of the piston skirt is taken as 0.65 to 0.8 times the cylinder bore. Now the total length of the piston (L) is given by

$$L = \text{Length of skirt} + \text{Length of ring section} + \text{Top land}$$

The length of the piston usually varies between D and 1.5 D. It may be noted that a longer piston provides better bearing surface for quiet running of the engine, but it should not be made unnecessarily long as it will increase its own mass and thus the inertia forces.

32.12 Piston Pin

The piston pin (also called gudgeon pin or wrist pin) is used to connect the piston and the connecting rod. It is usually made hollow and tapered on the inside, the smallest inside diameter being at the centre of the pin, as shown in Fig. 32.5.

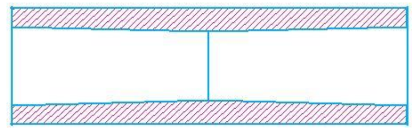


Fig.32.5. Piston pin.

The piston pin passes through the bosses provided on the inside of the piston skirt and the bush of the small end of the connecting rod. The centre of piston pin should be 0.02 D to 0.04 D above the centre of the skirt, in order to off-set the turning effect of the friction and to obtain uniform distribution of pressure between the piston and the cylinder liner.

The material used for the piston pin is usually case hardened steel alloy containing nickel, chromium, molybdenum or vanadium having tensile strength from 710 MPa to 910 MPa.

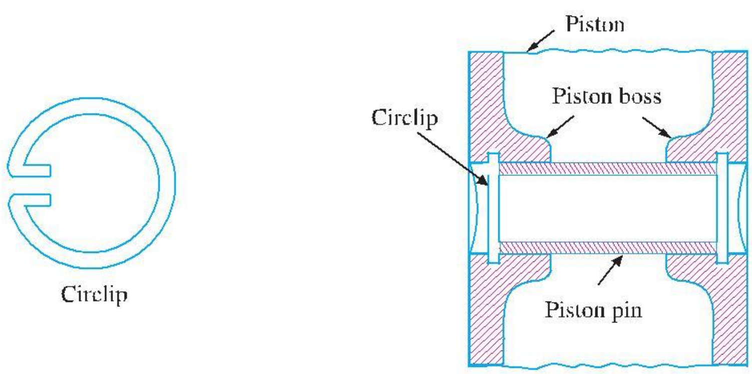


Fig. 32.6. Full floating type piston pin.

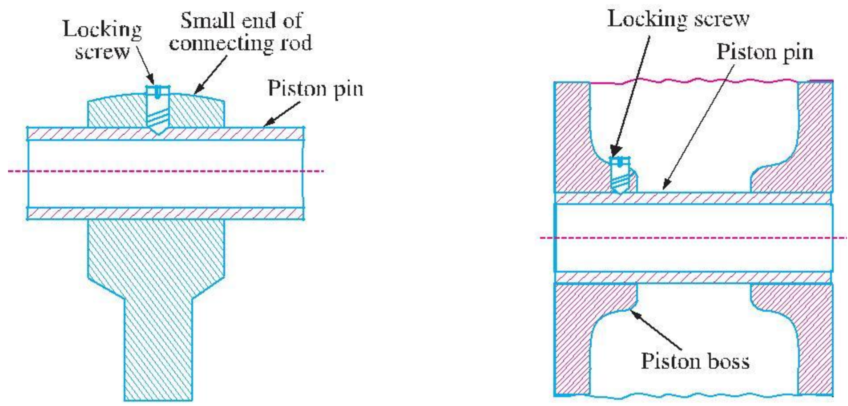
The connection between the piston pin and the small end of the connecting rod may be made either ***full floating type*** or ***semi-floating type.*** In the full floating type, the piston pin is free to turn both in the *piston bosses and the bush of the small end of the connecting rod. The end movements of the piston pin should be secured by means of spring circlips, as shown in Fig. 32.6, in order to prevent the pin from touching and scoring the cylinder liner.

In the semi-floating type, the piston pin is either free to turn in the piston bosses and rigidly secured to the small end of the connecting rod, or it is free to turn in the bush of the small end of the connecting rod and is rigidly secured in the piston bosses by means of a screw, as shown in Fig. 32.7

The piston pin should be designed for the maximum gas load or the inertia force of the piston, whichever is larger. The bearing area of the piston pin should be about equally divided between the piston pin bosses and the connecting rod bushing. Thus, the length of the pin in the connecting rod bushing will be about 0.45 of the cylinder bore or piston diameter (D), allowing for the end clearance

* The mean diameter of the piston bosses is made 1.4 d_0 for cast iron pistons and 1.5 d_0 for aluminium pistons, where d_0 is the outside diameter of the piston pin. The piston bosses are usually tapered, increasing the diameter towards the piston wall.

of the pin etc. The outside diameter of the piston pin (d_0) is determined by equating the load on the piston due to gas pressure (p) and the load on the piston pin due to bearing pressure (p_{b1}) at the small end of the connecting rod bushing.



(*a*) Piston pin secured to the small end of the connecting rod.

(*b*) Piston pin secured to the boss of the piston.

Fig. 32.7. Semi-floating type piston pin.

Let d_0 = Outside diameter of the piston pin in mm

l_1 = Length of the piston pin in the bush of the small end of the connecting rod in mm. Its value is usually taken as 0.45 D.

p_{b1} = Bearing pressure at the small end of the connecting rod bushing in N/mm^2. Its value for the bronze bushing may be taken as 25 N/mm^2.

We know that load on the piston due to gas pressure or gas load

$$= \frac{\pi D^2}{4} \cdot p \qquad \text{..}(i)$$

and load on the piston pin due to bearing pressure or bearing load

$$= \text{Bearing pressure} \times \text{Bearing area} = p_{b1} \times d_0 \times l_1 \qquad \text{...}(ii)$$

From equations **(*i*)** and **(*ii*)**, the outside diameter of the piston pin (d_0) may be obtained.

The piston pin may be checked in bending by assuming the gas load to be uniformly distributed over the length l_1 with supports at the centre of the bosses at the two ends. From Fig. 32.8, we find that the length between the supports,

$$l_2 = l_1 + \frac{D - l_1}{2} = \frac{l_1 + D}{2}$$

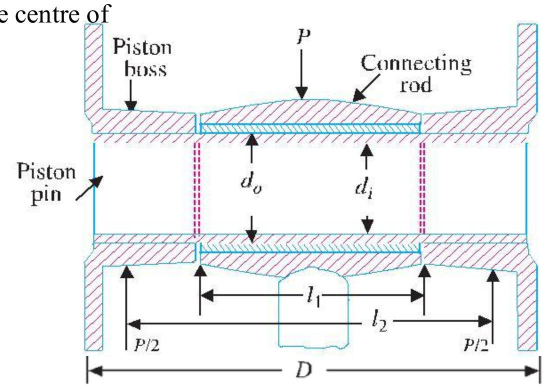


Fig. 32.8

Now maximum bending moment at the centre of the pin,

$$M = \frac{P}{2} \cdot \frac{l_2}{2} - \frac{P}{l_1} \cdot \frac{l_1}{2} \cdot \frac{l_1}{4}$$

$$= \frac{P}{2} \cdot \frac{l_2}{2} - \frac{P}{2} \cdot \frac{l_1}{4}$$

$$= \frac{P}{2}\left(\frac{l_1 + D}{2 \cdot 2}\right) - \frac{P}{2} \cdot \frac{l_1}{4}$$

$$= \frac{P.l_1}{8} + \frac{P.D}{8} - \frac{P.l_1}{8} = \frac{P.D}{8} \qquad \text{...}(iii)$$

We have already discussed that the piston pin is made hollow. Let d_0 and d_i be the outside and inside diameters of the piston pin. We know that the section modulus,

$$Z = \frac{\pi}{32}\left[\frac{(d_0)^4 - (d_i)^4}{d_0}\right]$$

We know that maximum bending moment,

$$M = Z \cdot \sigma_b = \frac{\pi}{32}\left[\frac{(d_0)^4 - (d_i)^4}{d_0}\right]\sigma_b$$

where σ_b = Allowable bending stress for the material of the piston pin. It is usually taken as 84 MPa for case hardened carbon steel and 140 MPa for heat treated alloy steel.

Assuming $d_i = 0.6\, d_0$, the induced bending stress in the piston pin may be checked.

Example 32.2. *Design a cast iron piston for a single acting four stroke engine for the following data:*

Cylinder bore = 100 mm ; Stroke = 125 mm ; Maximum gas pressure = 5 N/mm^2 ; Indicated mean effective pressure = 0.75 N/mm^2 ; Mechanical efficiency = 80% ; Fuel consumption = 0.15 kg per brake power per hour ; Higher calorific value of fuel = 42 × 10^3 kJ/kg ; Speed = 2000 r.p.m.

Any other data required for the design may be assumed.

Solution. Given : D = 100 mm ; L = 125 mm = 0.125 m ; p = 5 N/mm^2 ; p_m = 0.75 N/mm^2; η_m = 80% = 0.8 ; m = 0.15 kg / BP / h = 41.7 × 10^{-6} kg / BP / s; HCV = 42 × 10^3 kJ / kg ; N = 2000 r.p.m.

The dimensions for various components of the piston are determined as follows :

1. ***Piston head or crown***

The thickness of the piston head or crown is determined on the basis of strength as well as on the basis of heat dissipation and the larger of the two values is adopted.

We know that the thickness of piston head on the basis of strength,

$$t_H = \sqrt{\frac{3p \cdot D^2}{16\,\sigma_t}} = \sqrt{\frac{3 \cdot 5(100)^2}{16 \cdot 38}} = 15.7 \text{ say } 16 \text{ mm}$$

...(Taking σ_t for cast iron = 38 MPa = 38 N/mm^2)

Since the engine is a four stroke engine, therefore, the number of working strokes per minute, $n = N / 2 = 2000 / 2 = 1000$

and cross-sectional area of the cylinder,

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(100)^2 = 7855 \text{ mm}^2$$

We know that indicated power,

$$IP = \frac{p_m \cdot L \cdot A \cdot n}{60} = \frac{0.75 \times 0.125 \times 7855 \times 1000}{60} = 12\,270 \text{ W}$$

$$= 12.27 \text{ kW}$$

$\therefore$ Brake power, $BP = IP \times \eta_m = 12.27 \times 0.8 = 9.8$ kW ...($\because \eta_m = BP / IP$)

We know that the heat flowing through the piston head,

$$H = C \times HCV \times m \times BP$$

$$= 0.05 \times 42 \times 10^3 \times 41.7 \times 10^{-6} \times 9.8 = 0.86 \text{ kW} = 860 \text{ W}$$

....(Taking $C = 0.05$)

$\therefore$Thickness of the piston head on the basis of heat dissipation,

$$t_H = \frac{H}{12.56\,k\,(T_C - T_E)} = \frac{860}{12.56 \times 46.6 \times 220} = 0.0067 \text{ m} = 6.7 \text{ mm}$$

...($\because$ For cast iron , $k = 46.6$ W/m/°C, and $T_C - T_E = 220$°C)

Taking the larger of the two values, we shall adopt

$t_H = 16$ mm **Ans.**

Since the ratio of L / D is 1.25, therefore a cup in the top of the piston head with a radius equal to 0.7 D (*i.e.* 70 mm) is provided.

2. *Radial ribs*

The radial ribs may be four in number. The thickness of the ribs varies from $t_H / 3$ to $t_H / 2$.

$\therefore$ Thickness of the ribs, $t_R = 16 / 3$ to $16 / 2 = 5.33$ to 8 mm

Let us adopt $t_R = 7$ mm **Ans.**

3. *Piston rings*

Let us assume that there are total four rings (*i.e.* $n_r = 4$) out of which three are compression rings and one is an oil ring.

We know that the radial thickness of the piston rings,

$$t_1 = D\sqrt{\frac{3\,p_w}{\sigma_t}} = 100\sqrt{\frac{3 \times 0.035}{90}} = 3.4 \text{ mm}$$

...(Taking $p_w = 0.035$ N/mm^2, and $\sigma_t = 90$ MPa)

and axial thickness of the piston rings

$$t_2 = 0.7\,t_1 \text{ to } t_1 = 0.7 \times 3.4 \text{ to } 3.4 \text{ mm} = 2.38 \text{ to } 3.4 \text{ mm}$$

Let us adopt $t_2 = 3$ mm

We also know that the minimum axial thickness of the pistion ring,

$$t_2 = \frac{D}{10\, n_r} = \frac{100}{10 \cdot 4} = 2.5 \text{ mm}$$

Thus the axial thickness of the piston ring as already calculated (*i.e.* $t_2 = 3$ mm)is satisfactory. **Ans.** The distance from the top of the piston to the first ring groove, *i.e.* the width of the top land,

$$b_1 = t_H \text{ to } 1.2\, t_H = 16 \text{ to } 1.2 \times 16 \text{ mm} = 16 \text{ to } 19.2 \text{ mm}$$

and width of other ring lands,

$$b_2 = 0.75\, t_2 \text{ to } t_2 = 0.75 \times 3 \text{ to } 3 \text{ mm} = 2.25 \text{ to } 3 \text{ mm}$$

Let us adopt $b_1 = 18$ mm ; and $b_2 = 2.5$ mm **Ans.**

We know that the gap between the free ends of the ring,

$$G_1 = 3.5\, t_1 \text{ to } 4\, t_1 = 3.5 \times 3.4 \text{ to } 4 \times 3.4 \text{ mm} = 11.9 \text{ to } 13.6 \text{ mm}$$

and the gap when the ring is in the cylinder,

$$G_2 = 0.002\, D \text{ to } 0.004\, D = 0.002 \times 100 \text{ to } 0.004 \times 100 \text{ mm} = 0.2 \text{ to } 0.4 \text{ mm}$$

Let us adopt $G_1 = 12.8$ mm ; and $G_2 = 0.3$ mm **Ans.**

4. ***Piston barrel***

Since the radial depth of the piston ring grooves (b) is about 0.4 mm more than the radial thickness of the piston rings (t_1), therefore,

$$b = t_1 + 0.4 = 3.4 + 0.4 = 3.8 \text{ mm}$$

We know that the maximum thickness of barrel,

$$t_3 = 0.03\, D + b + 4.5 \text{ mm} = 0.03 \times 100 + 3.8 + 4.5 = 11.3 \text{ mm}$$

and piston wall thickness towards the open end,

$$t_4 = 0.25\, t_3 \text{ to } 0.35\, t_3 = 0.25 \times 11.3 \text{ to } 0.35 \times 11.3 = 2.8 \text{ to } 3.9 \text{ mm}$$

Let us adopt $t_4 = 3.4$ mm

5. ***Piston skirt***

Let l = Length of the skirt in mm.

We know that the maximum side thrust on the cylinder due to gas pressure (p),

$$R = \mu \cdot \frac{\pi D^2}{4} \cdot p = 0.1 \cdot \frac{\pi (100)^2}{4} \cdot 5 = 3928 \text{ N}$$

...(Taking $\mu = 0.1$)

We also know that the side thrust due to bearing pressure on the piston barrel (p_b),

$$R = p_b \times D \times l = 0.45 \times 100 \times l = 45\, l \text{ N}$$

...(Taking $p_b = 0.45$ N/mm^2)

From above, we find that

$45\, l = 3928$ or $l = 3928 / 45 = 87.3$ say 90 mm **Ans.**

$\therefore$ Total length of the piston ,

$$L = \text{Length of the skirt} + \text{Length of the ring section} + \text{Top land}$$

$$= l + (4\, t_2 + 3 b_2) + b_1$$

$$= 90 + (4 \times 3 + 3 \times 3) + 18 = 129 \text{ say } 130 \text{ mm } \textbf{Ans.}$$

6. ***Piston pin***

Let d_0 = Outside diameter of the pin in mm,

l_1 = Length of pin in the bush of the small end of the connecting rod in mm, and

p_{b1} = Bearing pressure at the small end of the connecting rod bushing in N/mm^2. It value for bronze bushing is taken as 25 N/mm^2.

We know that load on the pin due to bearing pressure

$$= \text{Bearing pressure} \times \text{Bearing area} = p_{b1} \times d_0 \times l_1$$

$$= 25 \times d_0 \times 0.45 \times 100 = 1125\, d_0 \text{ N} \qquad \text{...(Taking } l_1 = 0.45\, D)$$

We also know that maximum load on the piston due to gas pressure or maximum gas load

$$= \frac{\pi D^2}{4} \cdot p = \frac{\pi (100)^2}{4} \cdot 5 = 39\,275 \text{ N}$$

From above, we find that

$$1125\, d_0 = 39\,275 \text{ or } d_0 = 39\,275 / 1125 = 34.9 \text{ say } 35 \text{ mm } \textbf{Ans.}$$

The inside diameter of the pin (d_i) is usually taken as 0.6 d_0.

$$d_i = 0.6 \times 35 = 21 \text{ mm } \textbf{Ans.}$$

Let the piston pin be made of heat treated alloy steel for which the bending stress (σ_b)may be taken as 140 MPa. Now let us check the induced bending stress in the pin.

We know that maximum bending moment at the centre of the pin,

$$M = \frac{P \cdot D}{8} = \frac{39\,275 \cdot 100}{8} = 491 \cdot 10^3 \text{ N-mm}$$

We also know that maximum bending moment (M),

$$491 \times 10^3 = \frac{\pi}{32} \frac{(d_0)^4 - (d_i)^4}{d_0} \sigma_b = \frac{\pi}{32} \frac{(35)^4 - (21)^4}{35} \sigma_b = 3664\ \sigma_b$$

$$\therefore \quad \sigma_b = 491 \times 10^3 / 3664 = 134 \text{ N/mm}^2 \text{ or MPa}$$

Since the induced bending stress in the pin is less than the permissible value of 140 MPa (*i.e.* 140 N/mm^2), therefore, the dimensions for the pin as calculated above (*i.e.* d_0 = 35 mm and d_i = 21 mm) are satisfactory.

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32.13 Connecting Rod

The connecting rod is the intermediate member between the piston and the crankshaft. Its primary function is to transmit the push and pull from the piston pin to the crankpin and thus convert the reciprocating motion of the piston into the rotary motion of the crank. The usual form of the connecting rod in internal combustion engines is shown in Fig. 32.9. It consists of a long shank, a small end and a big end. The cross-section of the shank may be rectangular, circular, tubular, *I*-section or *H*-section. Generally circular section is used for low speed engines while *I*-section is preferred for high speed engines.

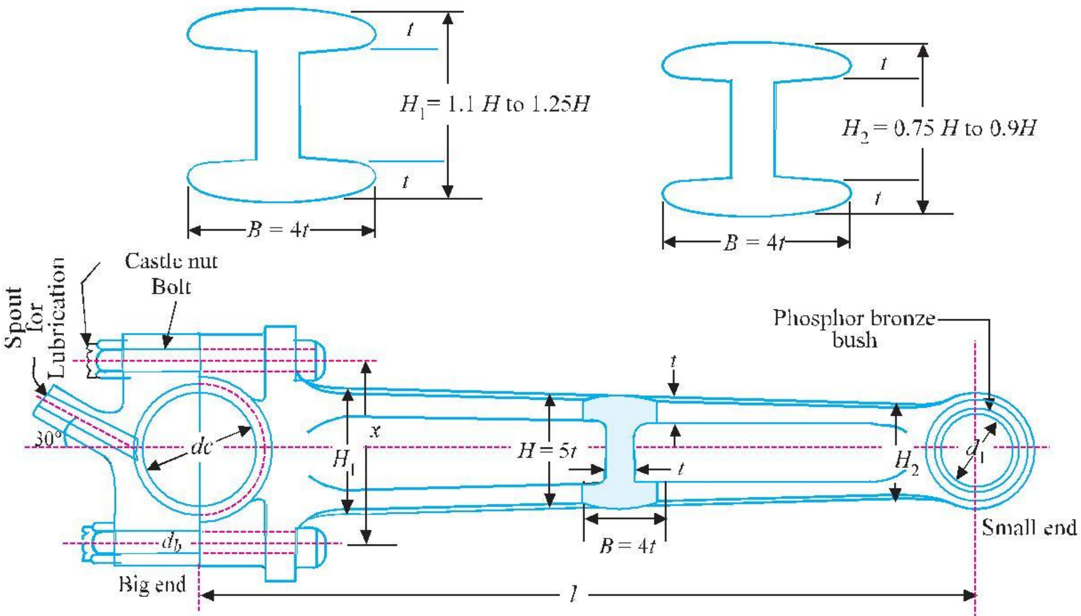

Fig. 32.9. Connecting rod.

The *length of the connecting rod (l) depends upon the ratio of l / r, where r is the radius of crank. It may be noted that the smaller length will decrease the ratio l / r. This increases the angularity of the connecting rod which increases the side thrust of the piston against the cylinder liner which in turn increases the wear of the liner. The larger length of the connecting rod will increase the ratio l / r. This decreases the angularity of the connecting rod and thus decreases the side thrust and the resulting wear of the cylinder. But the larger length of the connecting rod increases the overall height of the engine. Hence, a compromise is made and the ratio l / r is generally kept as 4 to 5.

The small end of the connecting rod is usually made in the form of an eye and is provided with a bush of phosphor bronze. It is connected to the piston by means of a piston pin.

The big end of the connecting rod is usually made split (in two **halves) so that it can be mounted easily on the crankpin bearing shells. The split cap is fastened to the big end with two cap bolts. The bearing shells of the big end are made of steel, brass or bronze with a thin lining (about 0.75 mm) of white metal or babbit metal. The wear of the big end bearing is allowed for by inserting thin metallic strips (known as ***shims***) about 0.04 mm thick between the cap and the fixed half of the connecting rod. As the wear takes place, one or more strips are removed and the bearing is trued up.

* It is the distance between the centres of small end and big end of the connecting rod.

** One half is fixed with the connecting rod and the other half (known as cap) is fastened with two cap bolts.

The connecting rods are usually manufactured by drop forging process and it should have adequate strength, stiffness and minimum weight. The material mostly used for connecting rods varies from mild carbon steels (having 0.35 to 0.45 percent carbon) to alloy steels (chrome-nickel or chrome-molybdenum steels). The carbon steel having 0.35 percent carbon has an ultimate tensile strength of about 650 MPa when properly heat treated and a carbon steel with 0.45 percent carbon has a ultimate tensile strength of 750 MPa. These steels are used for connecting rods of industrial engines. The alloy steels have an ultimate tensile strength of about 1050 MPa and are used for connecting rods of aeroengines and automobile engines.

The bearings at the two ends of the connecting rod are either splash lubricated or pressure lubricated. The big end bearing is usually splash lubricated while the small end bearing is pressure lubricated. In the **splash lubrication system**, the cap at the big end is provided with a dipper or spout and set at an angle in such a way that when the connecting rod moves downward, the spout will dip into the lubricating oil contained in the sump. The oil is forced up the spout and then to the big end bearing. Now when the connecting rod moves upward, a splash of oil is produced by the spout. This splashed up lubricant find its way into the small end bearing through the widely chamfered holes provided on the upper surface of the small end.

In the **pressure lubricating system,** the lubricating oil is fed under pressure to the big end bearing through the holes drilled in crankshaft, crankwebs and crank pin. From the big end bearing, the oil is fed to small end bearing through a fine hole drilled in the shank of the connecting rod. In some cases, the small end bearing is lubricated by the oil scrapped from the walls of the cyinder liner by the oil scraper rings.

32.14 Forces Acting on the Connecting Rod

The various forces acting on the connecting rod are as follows :

1. Force on the piston due to gas pressure and inertia of the reciprocating parts,
2. Force due to inertia of the connecting rod or inertia bending forces,
3. Force due to friction of the piston rings and of the piston, and
4. Force due to friction of the piston pin bearing and the crankpin bearing.

We shall now derive the expressions for the forces acting on a vertical engine, as discussed below.

1. ***Force on the piston due to gas pressure and inertia of reciprocating parts***

Consider a connecting rod *PC* as shown in Fig. 32.10.

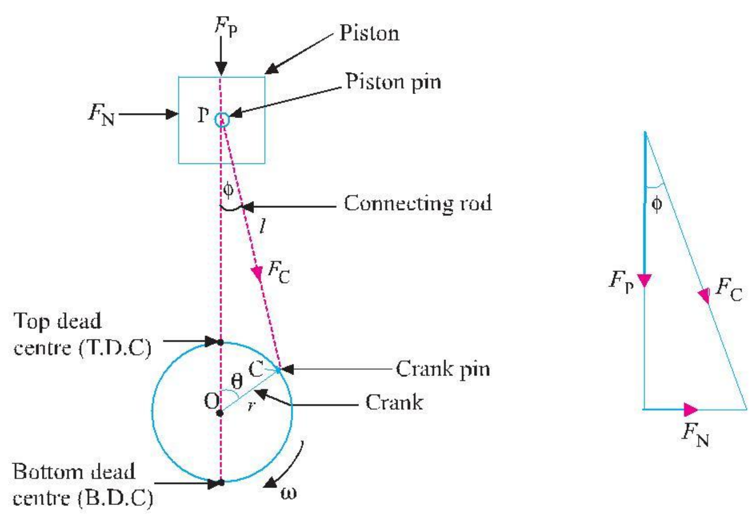

Fig. 32.10. Forces on the connecting rod.

1. Induction : turning rotor sucks in mixture of petrol and air.

2. Compression: Fuel-air mixture is compressed as rotor carriers it round.

3. Ignition: Compressed fuel-air mixture is ignited by the spark plug.

4. Exhuast : the rotor continues to turn and pushed out waste gases.

Let

p = Maximum pressure of gas,

D = Diameter of piston,

A = Cross-section area of piston = $\dfrac{\pi D^2}{4}$,

m_R = Mass of reciprocating parts,

= Mass of piston, gudgeon pin etc. + $\frac{1}{3}$ rd mass of connecting rod,

ω = Angular speed of crank,

φ = Angle of inclination of the connecting rod with the line of stroke, θ = Angle of inclination of the crank from top dead centre,

r = Radius of crank,

l = Length of connecting rod, and

n = Ratio of length of connecting rod to radius of crank = l / r.

We know that the force on the piston due to pressure of gas,

$$F_L = \text{Pressure} \times \text{Area} = p \, . \, A = p \times \pi D^2 / 4$$

and inertia force of reciprocating parts,

$$F_I = \text{Mass} \times {}^*\text{Acceleration} = m_R \, . \, \omega^2 \, . \, r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

It may be noted that the inertia force of reciprocating parts opposes the force on the piston when it moves during its downward stroke (*i. e.* when the piston moves from the top dead centre to bottom dead centre). On the other hand, the inertia force of the reciprocating parts helps the force on the piston when it moves from the bottom dead centre to top dead centre.

∴ Net force acting on the piston or piston pin (or gudgeon pin or wrist pin),

$$F_P = \text{Force due to gas pressure} \mp \text{Inertia force}$$

$$= F_L \mp F_I$$

The –ve sign is used when piston moves from TDC to BDC and +ve sign is used when piston moves from BDC to TDC.

When weight of the reciprocating parts ($W_R = m_R \, . \, g$) is to be taken into consideration, then

$$F_P = F_L \mp F_I \pm W_R$$

---

* Acceleration of reciprocating parts = $\omega^2 r \left(\cos\theta + \dfrac{\cos 2\theta}{n} \right)$

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The force F_P gives rise to a force F_C in the connecting rod and a thrust F_N on the sides of the cylinder walls. From Fig. 32.10, we see that force in the connecting rod at any instant,

$$F_C = \frac{F_P}{\cos \varphi} = \frac{F_P}{\sqrt{1 - \dfrac{\sin^2 \theta}{n^2}}}$$

The force in the connecting rod will be maximum when the crank and the connecting rod are perpendicular to each other (*i.e.* when $\theta = 90°$). But at this position, the gas pressure would be decreased considerably. ***Thus, for all practical purposes, the force in the connecting rod*** (F_C) ***is taken equal to the maximum force on the piston due to pressure of gas*** (F_L), neglecting piston inertia effects.

2. ***Force due to inertia of the connecting rod or inertia bending forces***

Consider a connecting rod *PC* and a crank *OC* rotating with uniform angular velocity ω rad / s. In order to find the accleration of various points on the connecting rod, draw the Klien's acceleration diagram *CQNO* as shown in Fig. 32.11 (*a*). *CO* represents the acceleration of *C* towards *O* and *NO* represents the accleration of *P* towards *O.* The acceleration of other points such as *D, E, F* and *G* etc., on the connecting rod *PC* may be found by drawing horizontal lines from these points to intresect *CN* at *d, e, f,* and *g* respectively. Now *dO, eO, fO* and *gO* respresents the acceleration of *D, E, F* and *G* all towards *O.* The inertia force acting on each point will be as follows:

Inertia force at $C = m \times \omega^2 \times CO$

Inertia force at $D = m \times \omega^2 \times dO$

Inertia force at $E = m \times \omega^2 \times eO$, and so on.

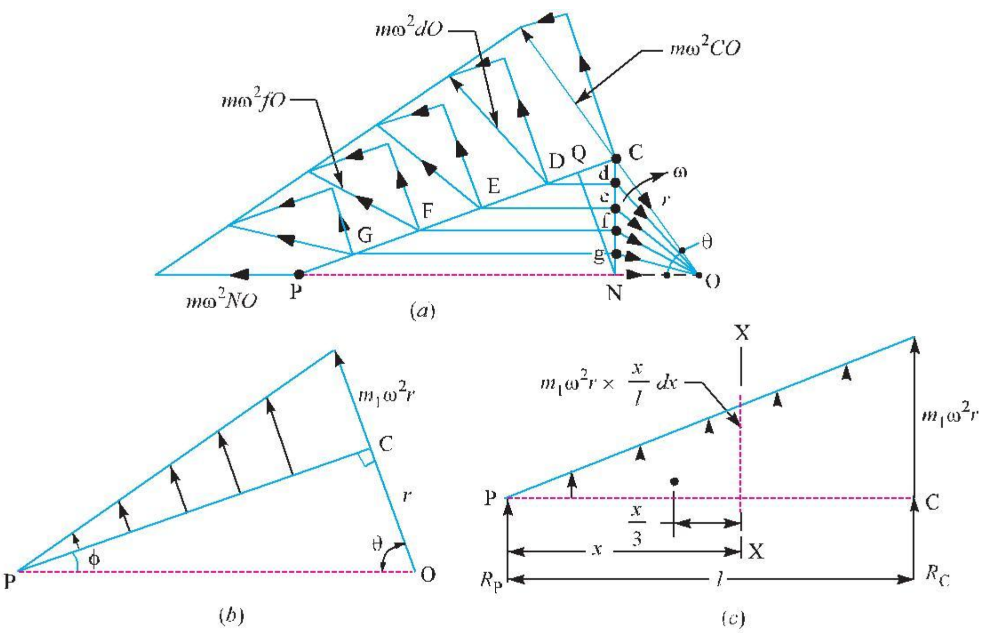


Fig. 32.11. Inertia bending forces.

The inertia forces will be opposite to the direction of acceleration or centrifugal forces. The inertia forces can be resolved into two components, one parallel to the connecting rod and the other perpendicular to rod. The parallel (or longitudinal) components adds up algebraically to the force

* For derivation, please refer ot Authors' popular book on **'Theory of Machines'.**

acting on the connecting rod (F_C) and produces thrust on the pins. The perpendicular (or transverse) components produces bending action (also called whipping action) and the stress induced in the connecting rod is called ***whipping stress.***

It may be noted that the perpendicular components will be maximum, when the crank and connecting rod are at right angles to each other.

The variation of the inertia force on the connecting rod is linear and is like a simply supported beam of variable loading as shown in Fig. 32.11 (*b*) and (*c*). Assuming that the connecting rod is of uniform cross-section and has mass m_1 kg per unit length, therefore,

Inertia force per unit length at the crankpin

$$= m_1 \times \omega^2 r$$

and inertia force per unit length at the piston pin = 0

Inertia force due to small element of length dx at a distance x from the piston pin P,

$$dF_1 = m_1 \times \omega^2 r \times \frac{x}{l} \times dx$$

$\therefore$ Resultant inertia force,

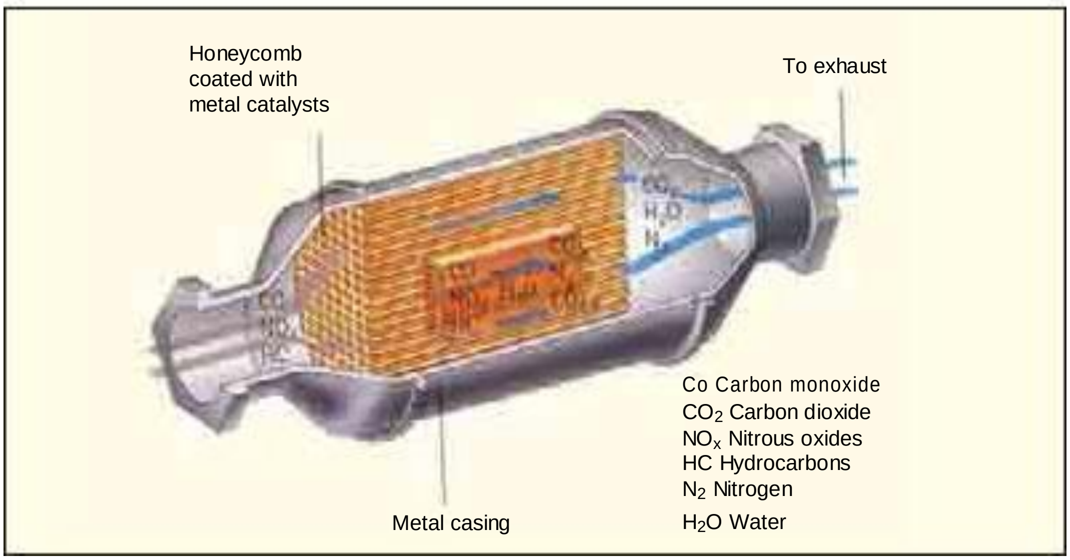
$$F_I = \int_0^l m_1 \cdot \omega^2 r \cdot \frac{x}{l} \cdot dx = \frac{m_1 \cdot \omega^2 r}{l} \left[\frac{x^2}{2}\right]_0^l$$

$$= \frac{m_1 . l}{2} \cdot \omega^2 r = \frac{m}{2} \cdot \omega^2 r \qquad \text{...(Substituting } m_1 \cdot l = m)$$

This resultant inertia force acts at a distance of $2l / 3$ from the piston pin P.

Since it has been assumed that $\frac{1}{3}$ rd mass of the connecting rod is concentrated at piston pin P (*i.e.* small end of connecting rod) and $\frac{2}{3}$ rd at the crankpin (*i.e.* big end of connecting rod), therefore, the reaction at these two ends will be in the same proportion.

$$i.e. \quad R_P = \frac{1}{3} F_I, \text{ and } R_C = \frac{2}{3} F_I$$



Emissions of an automobile.

Now the bending moment acting on the rod at section $X - X$ at a distance x from P,

$$M_X = R_P \cdot x - {}^*m_1 \cdot \omega^2 r \cdot \frac{x}{l} \cdot \frac{1}{2} x \cdot \frac{x}{3}$$

$$= \frac{1}{3} F_I \cdot x - \frac{m_1 \cdot l}{2} \cdot \omega^2 r \cdot \frac{x^3}{3l^2}$$

...(Multiplying and dividing the latter expression by l)

$$= \frac{F_I \cdot x}{3} - F_I \cdot \frac{x^3}{3l^2} = \frac{F_I}{3}\left(x - \frac{x^3}{l^2}\right) \quad ...(i)$$

For maximum bending moment, differentiate M_X with respect to x and equate to zero, *i.e.*

$$\frac{dM_X}{dx} = 0 \text{ or } \frac{F_I}{3}\left(1 - \frac{3x^2}{l^2}\right) = 0$$

$$\therefore \quad 1 - \frac{3x^2}{l^2} = 0 \text{ or } 3x^2 = l^2 \text{ or } x = \frac{l}{\sqrt{3}}$$

Maximum bending moment,

$$M_{max} = \frac{F_I}{3}\left[\frac{l}{\sqrt{3}} - \frac{\left(\frac{l}{\sqrt{3}}\right)^3}{l^2}\right] \quad ...[\text{From equation } (i)]$$

$$= \frac{F_I}{3}\left[\frac{l}{\sqrt{3}} - \frac{l}{3\sqrt{3}}\right] = \frac{F_I \cdot l}{3\sqrt{3}} \cdot \frac{2}{3} = \frac{2F_I \cdot l}{9\sqrt{3}}$$

$$= 2 \cdot \frac{m}{2} \cdot \omega^2 r \cdot \frac{l}{9\sqrt{3}} = m \cdot \omega^2 r \cdot \frac{l}{9\sqrt{3}}$$

and the maximum bending stress, due to inertia of the connecting rod,

$$\sigma_{max} = \frac{M_{max}}{Z}$$

where Z = Section modulus.

From above we see that the maximum bending moment varies as the square of speed, therefore, the bending stress due to high speed will be dangerous. It may be noted that the maximum axial force and the maximum bending stress do not occur simultaneously. In an I.C. engine, the maximum gas load occurs close to top dead centre whereas the maximum bending stress occurs when the crank angle θ = 65° to 70° from top dead centre. The pressure of gas falls suddenly as the piston moves from dead centre. ***Thus the general practice is to design a connecting rod by assuming the force in the connecting rod (F_C) equal to the maximum force due to pressure (F_L), neglecting piston inertia effects and then checked for bending stress due to inertia force (i.e. whipping stress).***

3. ***Force due to friction of piston rings and of the piston***

The frictional force (F) of the piston rings may be determined by using the following expression :

$$F = \pi D \cdot t_R \cdot n_R \cdot p_R \cdot \mu$$

where D = Cylinder bore,

t_R = Axial width of rings,

* B.M. due to variable force from 0 to $m_1 \omega^2 r \cdot \frac{x}{l}$ is equal to the area of triangle multiplied by the distance of C.G. from $X - X$ *i.e.* $\frac{x}{3}$.

n_R = Number of rings,

p_R = Pressure of rings (0.025 to 0.04 N/mm^2),

and μ = Coefficient of friction (about 0.1).

Since the frictional force of the piston rings is usually very small, therefore, it may be neglected.

The friction of the piston is produced by the normal component of the piston pressure which varies from 3 to 10 percent of the piston pressure. If the coefficient of friction is about 0.05 to 0.06, then the frictional force due to piston will be about 0.5 to 0.6 of the piston pressure, which is very low. Thus, the frictional force due to piston is also neglected.

4. ***Force due to friction of the piston pin bearing and crankpin bearing***

The force due to friction of the piston pin bearing and crankpin bearing, is to bend the connecting rod and to increase the compressive stress on the connecting rod due to the direct load. Thus, the maximum compressive stress in the connecting rod will be

$\sigma_{c\,(max)}$ = Direct compressive stress + Maximum bending or whipping stress due to inertia bending stress

32.15 Design of Connecting Rod

In designing a connecting rod, the following dimensions are required to be determined :

1. Dimensions of cross-section of the connecting rod,

2. Dimensions of the crankpin at the big end and the piston pin at the small end,

3. Size of bolts for securing the big end cap, and

4. Thickness of the big end cap.

The procedure adopted in determining the above mentioned dimensions is discussed as below :

This experimental car burns hydrogen fuel in an ordinary piston engine. Its exhaust gases cause no pollution, because they contain only water vapour.

1. ***Dimensions of cross-section of the connecting rod***

A connecting rod is a machine member which is subjected to alternating direct compressive and tensile forces. Since the compressive forces are much higher than the tensile forces, therefore, the cross-section of the connecting rod is designed as a strut and the Rankine's formula is used.

A connecting rod, as shown in Fig. 32.12, subjected to an axial load W may buckle with X-axis as neutral axis (*i.e.* in the plane of motion of the connecting rod) or Y-axis as neutral axis (*i.e.* in the plane perpendicular to the plane of motion). The connecting rod is considered like both ends hinged for buckling about X-axis and both ends fixed for buckling about Y-axis.

A connecting rod should be equally strong in buckling about both the axes.

Let A = Cross-sectional area of the connecting rod,

l = Length of the connecting rod,

σ_c = Compressive yield stress,

W_B = Buckling load,

I_{xx} and I_{yy} = Moment of inertia of the section about X-axis and Y-axis respectively, and

k_{xx} and k_{yy} = Radius of gyration of the section about X-axis and Y-axis respectively.

According to Rankine's formula,

$$W_B \text{ about } X\text{-axis} = \frac{\sigma_c . A}{1 + a\left(\frac{L}{k_{xx}}\right)^2} = \frac{\sigma_c . A}{1 + a\left(\frac{l}{k_{xx}}\right)^2} \qquad \ldots(\because \text{For both ends hinged, } L = l)$$

and

$$W_B \text{ about } Y\text{-axis} = \frac{\sigma_c . A}{1 + a\left(\frac{L}{k_{yy}}\right)^2} = \frac{\sigma_c . A}{1 + a\left(\frac{l}{2k_{yy}}\right)^2} \qquad \ldots\left[\because \text{For both ends fixed, } L = \frac{l}{2}\right]$$

where L = Equivalent length of the connecting rod, and

a = Constant

= 1 / 7500, for mild steel

= 1 / 9000, for wrought iron

= 1 / 1600, for cast iron

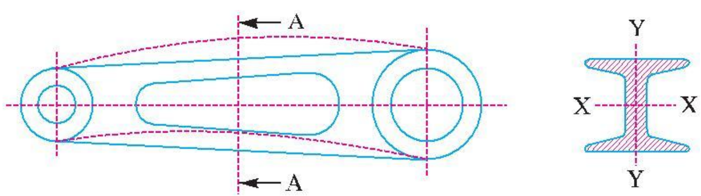


Fig. 32.12. Buckling of connecting rod.

In order to thave a connecting rod equally strong in buckling about both the axes, the buckling loads must be equal, *i.e.*

$$\frac{\sigma_c . A}{1 + a\left(\frac{l}{k_{xx}}\right)^2} = \frac{\sigma_c . A}{1 + a\left(\frac{l}{2k_{yy}}\right)^2} \quad \text{or} \quad \left(\frac{l}{k_{xx}}\right)^2 = \left(\frac{l}{2k_{yy}}\right)^2$$

$$\therefore \quad k_{xx}^2 = 4\, k_{yy}^2 \quad \text{or} \quad I_{xx} = 4\, I_{yy} \qquad \ldots(\because I = A \cdot k^2)$$

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This shows that the connecting rod is four times strong in buckling about Y-axis than about X-axis. If $I_{xx} > 4\, I_{yy}$, then buckling will occur about Y- axis and if $I_{xx} < 4\, I_{yy}$, buckling will occur about X-axis. In actual practice, I_{xx} is kept slightly less than $4\, I_{yy}$. It is usually taken between 3 and 3.5 and the connecting rod is designed for bucking about X-axis. The design will always be satisfactory for buckling about Y-axis.

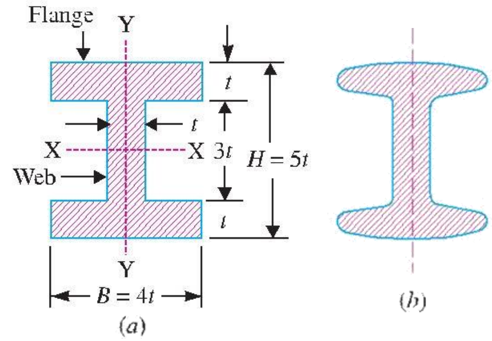


Fig. 32.13. *I*-section of connecting rod.

The most suitable section for the connecting rod is I-section with the proportions as shown in Fig. 32.13 (*a*).

Let thickness of the flange and web of the section $= t$

Width of the section, $B = 4\, t$

and depth or height of the section,

$$H = 5t$$

From Fig. 32.13 (*a*), we find that area of the section,

$$A = 2\,(4\,t \times t) + 3\,t \times t = 11\,t^2$$

Moment of inertia of the section about X-axis,

$$I_{xx} = \frac{1}{12}\left[4\,t\,(5\,t)^3 - 3\,t\,(3\,t)^3\right] = \frac{419}{12}\,t^4$$

and moment of inertia of the section about Y-axis,

$$I_{yy} = \left[2 \times \frac{1}{12}\,t \times (4t)^3 + \frac{1}{12}\,(3t)\,t^3\right] = \frac{131}{12}\,t^4$$

$$\therefore \qquad \frac{I_{xx}}{I_{yy}} = \frac{419}{12} \times \frac{12}{131} = 3.2$$

Since the value of $\frac{I_{xx}}{I_{yy}}$ lies between 3 and 3.5, therefore, I-section chosen is quite satisfactory.

After deciding the proportions for I-section of the connecting rod, its dimensions are determined by considering the buckling of the rod about X-axis (assuming both ends hinged) and applying the Rankine's formula. We know that buckling load,

$$W_B = \frac{\sigma_c \cdot A}{1 + a\left(\frac{L}{k_{xx}}\right)^2}$$

The buckling load (W_B) may be calculated by using the following relation,

i.e. W_B = Max. gas force × Factor of safety

The factor of safety may be taken as 5 to 6.

Notes : (*a*) The I-section of the connecting rod is used due to its lightness and to keep the inertia forces as low as possible specially in case of high speed engines. It can also withstand high gas pressure.

(*b*) Sometimes a connecting rod may have rectangular section. For slow speed engines, circular section may be used.

(*c*) Since connecting rod is manufactured by forging, therefore the sharp corner of I-section are rounded off as shown in Fig. 32.13 (*b*) for easy removal of the section from dies.

The dimensions $B = 4\,t$ and $H = 5\,t$, as obtained above by applying the Rankine's formula, are at the middle of the connecting rod. The width of the section (B) is kept constant throughout the length of the connecting rod, but the depth or height varies. The depth near the small end (or piston end) is taken as $H_1 = 0.75\,H$ to $0.9H$ and the depth near the big end (or crank end) is taken $H_2 = 1.1H$ to $1.25H$.

2. *Dimensions of the crankpin at the big end and the piston pin at the small end*

Since the dimensions of the crankpin at the big end and the piston pin (also known as gudgeon pin or wrist pin) at the small end are limited, therefore, fairly high bearing pressures have to be allowed at the bearings of these two pins.

The crankpin at the big end has removable precision bearing shells of brass or bronze or steel with a thin lining (1 mm or less) of bearing metal (such as tin, lead, babbit, copper, lead) on the inner surface of the shell. The allowable bearing pressure on the crankpin depends upon many factors such as material of the bearing, viscosity of the lubricating oil, method of lubrication and the space limitations.The value of bearing pressure may be taken as 7 N/mm^2 to 12.5 N/mm^2 depending upon the material and method of lubrication used.

Engine of a motorcyle.

The piston pin bearing is usually a phosphor bronze bush of about 3 mm thickness and the allowable bearing pressure may be taken as 10.5 N/mm^2 to 15 N/mm^2.

Since the maximum load to be carried by the crankpin and piston pin bearings is the maximum force in the connecting rod (F_C), therefore the dimensions for these two pins are determined for the maximum force in the connecting rod (F_C) which is taken equal to the maximum force on the piston due to gas pressure (F_L) neglecting the inertia forces.

We know that maximum gas force,

$$F_L = \frac{\pi D^2}{4} \cdot p \qquad \ldots(i)$$

where D = Cylinder bore or piston diameter in mm, and

p = Maximum gas pressure in N/mm^2

Now the dimensions of the crankpin and piston pin are determined as discussed below

: Let d_c = Diameter of the crank pin in mm,

l_c = Length of the crank pin in mm,

p_{bc} = Allowable bearing pressure in N/mm^2, and

d_p, l_p and p_{bp} = Corresponding values for the piston pin,

We know that load on the crank pin

= Projected area × Bearing pressure

$= d_c \,.\, l_c \,.\, p_{bc}$ ...***(ii)*** Similarly, load on the piston pin

$= d_p \,.\, l_p \,.\, p_{bp}$ ...*(iii)*

Equating equations ***(i)*** and ***(ii)***, we have

$$F_L = d_c \cdot l_c \cdot p_{bc}$$

Taking $l_c = 1.25\, d_c$ to $1.5\, d_c$, the value of d_c and l_c are determined from the above expression. Again, equating equations ***(i)*** and ***(iii)***, we have

$$F_L = d_p \cdot l_p \cdot p_{bp}$$

Taking $l_p = 1.5\, d_p$ to $2\, d_p$, the value of d_p and l_p are determined from the above expression.

3. ***Size of bolts for securing the big end cap***

The bolts and the big end cap are subjected to tensile force which corresponds to the inertia force of the reciprocating parts at the top dead centre on the exhaust stroke. We know that inertia force of the reciprocating parts,

$$\therefore \quad F_I = m_R \cdot \omega^2 \cdot r \left(\cos\theta + \frac{\cos 2\theta}{l/r}\right)$$

We also know that at the top dead centre, the angle of inclination of the crank with the line of stroke, $\theta = 0$

$$\therefore \quad F_I = m_R \cdot \omega^2 \cdot r \left(1 + \frac{r}{l}\right)$$

where m_R = Mass of the reciprocating parts in kg,

ω = Angular speed of the engine in rad / s,

r = Radius of the crank in metres, and

l = Length of the connecting rod in metres.

The bolts may be made of high carbon steel or nickel alloy steel. Since the bolts are under repeated stresses but not alternating stresses, therefore, a factor of safety may be taken as 6.

Let d_{cb} = Core diameter of the bolt in mm,

σ_t = Allowable tensile stress for the material of the bolts in MPa, and

n_b = Number of bolts. Generally two bolts are used.

$\therefore$ Force on the bolts

$$= \frac{\pi}{4}(d_{cb})^2 \sigma_t \cdot n_b$$

Equating the inertia force to the force on the bolts, we have

$$F_I = \frac{\pi}{4}(d_{cb})^2 \sigma_t \cdot n_b$$

From this expression, d_{cb} is obtained. The nominal or major diameter (d_b) of the bolt is given by

$$d_b = \frac{d_{cb}}{0.84}$$

4. ***Thickness of the big end cap***

The thickness of the big end cap (t_c) may be determined by treating the cap as a beam freely supported at the cap bolt centres and loaded by the inertia force at the top dead centre on the exhaust stroke (*i.e.* F_I when $\theta = 0$). This load is assumed to act in between the uniformly distributed load and the centrally concentrated load. Therefore, the maximum bending moment acting on the cap will be taken as

$$M_C = {}^*\frac{F_I \cdot x}{6}$$

where x = Distance between the bolt centres.

= Dia. of crankpin or big end bearing (d_c) + 2 × Thickness of bearing liner (3 mm) + Clearance (3 mm)

Let b_c = Width of the cap in mm. It is equal to the length of the crankpin or big end bearing (l_c), and

σ_b = Allowable bending stress for the material of the cap in MPa.

We know that section modulus for the cap,

$$Z_C = \frac{b_c (t_c)^2}{6}$$

$$\therefore \text{ Bending stress, } \quad \sigma_b = \frac{M_C}{Z_C} = \frac{F_I \cdot x}{6} \times \frac{6}{b_c (t_c)^2} = \frac{F_I \cdot x}{b_c (t_c)^2}$$

From this expression, the value of t_c is obtained.

Note: The design of connecting rod should be checked for whipping stress (*i.e.* bending stress due to inertia force on the connecting rod).

Example 32.3. *Design a connecting rod for an I.C. engine running at 1800 r.p.m. and developing a maximum pressure of 3.15 N/mm^2. The diameter of the piston is 100 mm ; mass of the reciprocating parts per cylinder 2.25 kg; length of connecting rod 380 mm; stroke of piston 190 mm and compression ratio 6 : 1. Take a factor of safety of 6 for the design. Take length to diameter ratio for big end bearing as 1.3 and small end bearing as 2 and the corresponding bearing pressures as 10 N/mm^2 and 15 N/mm^2. The density of material of the rod may be taken as 8000 kg/m^3 and the allowable stress in the bolts as 60 N/mm^2 and in cap as 80 N/mm^2. The rod is to be of I-section for which you can choose your own proportions.*

Draw a neat dimensioned sketch showing provision for lubrication. Use Rankine formula for which the numerator constant may be taken as 320 N/mm^2 and the denominator constant 1 / 7500.

* We know that the maximum bending moment for a simply or freely supported beam with a uniformly distributed load of F_I over a length x between the supports (In this case, x is the distance between the cap bolt centres) is $\frac{F_I \cdot x}{8}$. When the load F_I is assumed to act at the centre of the freely supported beam, then the maximum bending moment is $\frac{F_I \cdot x}{4}$. Thus the maximum bending moment in between these two bending moments *i.e.* $\frac{F_I \cdot x}{8}$ and $\frac{F_I \cdot x}{4}$ is $\frac{F_I \cdot x}{6}$.

Solution. Given : N = 1800 r.p.m. ; p = 3.15 N/mm^2 ; D = 100 mm ; m_R = 2.25 kg ; l = 380 mm = 0.38 m ; Stroke = 190 mm ; *Compression ratio = 6 : 1 ; $F.\ S.$ = 6.

The connecting rod is designed as discussed below :

1. ***Dimension of I- section of the connecting rod***

Let us consider an I-section of the connecting rod, as shown in Fig. 32.14 (a), with the following proportions :

Flange and web thickness of the section = t

Width of the section, $B = 4t$

and depth or height of the section,

$$H = 5t$$

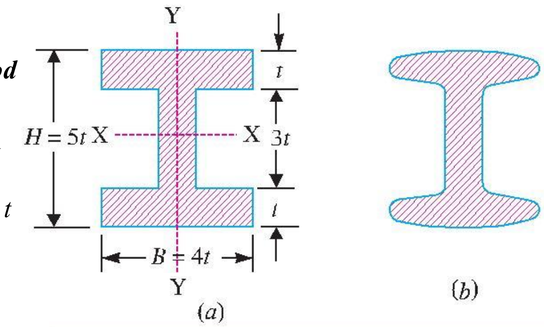


Fig. 32.14

First of all, let us find whether the section chosen is satisfactory or not.

We have already discussed that the connecting rod is considered like both ends hinged for buckling about X-axis and both ends fixed for buckling about Y-axis. The connecting rod should be equally strong in buckling about both the axes. We know that in order to have a connecting rod equally strong about both the axes,

$$I_{xx} = 4\,I_{yy}$$

where I_{xx} = Moment of inertia of the section about X-axis, and

I_{yy} = Moment of inertia of the section about Y-axis.

In actual practice, I_{xx} is kept slightly less than 4 I_{yy}. It is usually taken between 3 and 3.5 and the connecting rod is designed for buckling about X-axis.

Now, for the section as shown in Fig. 32.14 (a), area of the section,

$$A = 2\,(4\,t \times t\,) + 3t \times t = 11\,t^2$$

$$I_{xx} = \frac{1}{12}\left[4\,t\,(5t\,)^3 - 3\,t \cdot (3t\,)^3\right] = \frac{419}{12}\,t^4$$

and
$$I_{yy} = 2 \cdot \frac{1}{12} \cdot t(4t)^3 + \frac{1}{12} \cdot 3t \cdot t^3 = \frac{131}{12}\,t^4$$

$$\therefore \quad \frac{I_{xx}}{I_{yy}} = \frac{419}{12} \cdot \frac{12}{131} = 3.2$$

Since $\dfrac{I_{xx}}{I_{yy}}$ = 3.2, therefore the section chosen in quite satisfactory.

Now let us find the dimensions of this I-section. Since the connecting rod is designed by taking the force on the connecting rod (F_C) equal to the maximum force on the piston (F_L) due to gas pressure, therefore,

$$F_C = F_L = \frac{\pi\,D^2}{4} \cdot p = \frac{\pi(100)^2}{4} \cdot 3.15 = 24\ 740\ \text{N}$$

We know that the connecting rod is designed for buckling about X-axis (*i.e.* in the plane of motion of the connecting rod) assuming both ends hinged. Since a factor of safety is given as 6, therefore the buckling load,

$$W_B = F_C \times F.\ S. = 24\ 740 \times 6 = 148\ 440\ \text{N}$$

* Superfluous data

We know that radius of gyration of the section about X-axis,

$$k_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{419\, t^4}{12} \times \frac{1}{11\, t^2}} = 1.78\, t$$

Length of crank,

$$r = \frac{\text{Stroke of piston}}{2} = \frac{190}{2} = 95 \text{ mm}$$

Length of the connecting rod,

$$l = 380 \text{ mm} \qquad \text{...(Given)}$$

∴ Equivalent length of the connecting rod for both ends hinged,

$$L = l = 380 \text{ mm}$$

Now according to Rankine's formula, we know that buckling load (W_B),

$$148\,440 = \frac{\sigma_c \cdot A}{1 + a\left(\frac{L}{k_{xx}}\right)^2} = \frac{320 \times 11\, t^2}{1 + \frac{1}{7500}\left(\frac{380}{1.78\, t}\right)^2}$$

... (It is given that σ_c = 320 MPa or N/mm^2 and a = 1 / 7500)

$$\frac{148\,440}{320} = \frac{11\, t^2}{1 + \frac{6.1}{t^2}} = \frac{11\, t^4}{t^2 + 6.1}$$

$$464\,(t^2 + 6.1) = 11\, t^4$$

or $$t^4 - 42.2\, t^2 - 257.3 = 0$$

∴ $$t^2 = \frac{42.2 \pm \sqrt{(42.2)^2 + 4 \times 257.3}}{2} = \frac{42.2 \pm 53}{2} = 47.6$$

... (Taking +ve sign)

or $$t = 6.9 \text{ say } 7 \text{ mm}$$

Thus, the dimensions of I-section of the connecting rod are :

Thickness of flange and web of the section

$= t = 7$ mm **Ans.**

Width of the section, $B = 4\, t = 4 \times 7 = 28$ mm **Ans.**

and depth or height of the section,

$H = 5\, t = 5 \times 7 = 35$ mm **Ans.**

Piston and connecting rod.

These dimensions are at the middle of the connecting rod. The width (B) is kept constant throughout the length of the rod, but the depth (H) varies. The depth near the big end or crank end is kept as $1.1H$ to $1.25H$ and the depth near the small end or piston end is kept as $0.75H$ to $0.9H$. Let us take

Depth near the big end,

$$H_1 = 1.2H = 1.2 \times 35 = 42 \text{ mm}$$

and depth near the small end,

$$H_2 = 0.85H = 0.85 \times 35 = 29.75 \text{ say } 30 \text{ mm}$$

∴ Dimensions of the section near the big end

$$= 42 \text{ mm} \times 28 \text{ mm}$$

Ans. and dimensions of the section near the small end

$$= 30 \text{ mm} \times 28 \text{ mm} \textbf{ Ans.}$$

Since the connecting rod is manufactured by forging, therefore the sharp corners of I-section are rounded off, as shown in Fig. 32.14 (b), for easy removal of the section from the dies.

2. ***Dimensions of the crankpin or the big end bearing and piston pin or small end bearing***

Let d_c = Diameter of the crankpin or big end bearing,

l_c = length of the crankpin or big end bearing = $1.3\, d_c$ ...(Given)

p_{bc} = Bearing pressure = 10 N/mm^2 ...(Given)

We know that load on the crankpin or big end bearing

$$= \text{Projected area} \times \text{Bearing pressure}$$

$$= d_c \,.\, l_c \,.\, p_{bc} = d_c \times 1.3\, d_c \times 10 = 13\, (d_c)^2$$

Since the crankpin or the big end bearing is designed for the maximum gas force (F_L), therefore, equating the load on the crankpin or big end bearing to the maximum gas force, *i.e.*

$$13\, (d_c)^2 = F_L = 24\,740 \text{ N}$$

∴ $(d_c)^2 = 24\,740 / 13 = 1903$ or $d_c = 43.6$ say 44 mm **Ans.**

and $l_c = 1.3\, d_c = 1.3 \times 44 = 57.2$ say 58 mm **Ans.**

The big end has removable precision bearing shells of brass or bronze or steel with a thin lining (1mm or less) of bearing metal such as babbit.

Again, let d_p = Diameter of the piston pin or small end bearing,

l_p = Length of the piston pin or small end bearing = $2d_p$ ...(Given)

p_{bp} = Bearing pressure = 15 N/mm^2 ..(Given)

We know that the load on the piston pin or small end bearing

$$= \text{Project area} \times \text{Bearing pressure}$$

$$= d_p \,.\, l_p \,.\, p_{bp} = d_p \times 2\, d_p \times 15 = 30\, (d_p)^2$$

Since the piston pin or the small end bearing is designed for the maximum gas force (F_L), therefore, equating the load on the piston pin or the small end bearing to the maximum gas force, *i.e.*

$$30\, (d_p)^2 = 24\,740 \text{ N}$$

∴ $(d_p)^2 = 24\,740 / 30 = 825$ or $d_p = 28.7$ say 29 mm **Ans.**

and $l_p = 2\, d_p = 2 \times 29 = 58$ mm **Ans.**

The small end bearing is usually a phosphor bronze bush of about 3 mm thickness.

3. ***Size of bolts for securing the big end cap***

Let d_{cb} = Core diameter of the bolts,

σ_t = Allowable tensile stress for the material of the bolts

$= 60 \text{ N/mm}^2$ ...(Given)

and n_b = Number of bolts. Generally two bolts are used.

We know that force on the bolts

$$= \frac{\pi}{4}(d_{cb})^2 \sigma_t \cdot n_b = \frac{\pi}{4}(d_{cb})^2\, 60 \cdot 2 = 94.26\,(d_{cb})^2$$

The bolts and the big end cap are subjected to tensile force which corresponds to the inertia force of the reciprocating parts at the top dead centre on the exhaust stroke. We know that inertia force of the reciprocating parts,

$$F_I = m_R \cdot \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{l/r}\right)$$

We also know that at top dead centre on the exhaust stroke, θ = 0.

$$\therefore \quad F_I = m_R \cdot \omega^2 \cdot r\left(1 + \frac{r}{l}\right) = 2.25\left(\frac{2\pi \cdot 1800}{60}\right)^2 0.095\left(1 + \frac{0.095}{0.38}\right) \text{ N}$$

$$= 9490 \text{ N}$$

Equating the inertia force to the force on the bolts, we have

$$9490 = 94.26\,(d_{cb})^2 \text{ or } (d_{cb})^2 = 9490 / 94.26 = 100.7$$

$$\therefore \quad d_{cb} = 10.03 \text{ mm}$$

and nominal diameter of the bolt,

$$d_b = \frac{d_{cb}}{0.84} = \frac{10.03}{0.84} = 11.94 \text{ say } 12 \text{ mm } \textbf{Ans.}$$

4. ***Thickness of the big end cap***

Let t_c = Thickness of the big end cap,

b_c = Width of the big end cap. It is taken equal to the length of the crankpin or big end bearing (l_c)

= 58 mm (calculated above)

σ_b = Allowable bending stress for the material of the cap

$= 80 \text{ N/mm}^2$ ...(Given)

The big end cap is designed as a beam freely supported at the cap bolt centres and loaded by the inertia force at the top dead centre on the exhaust stroke (*i.e.* F_I when θ = 0). Since the load is assumed to act in between the uniformly distributed load and the centrally concentrated load, therefore, maximum bending moment is taken as

$$M_C = \frac{F_I \cdot x}{6}$$

where x = Distance between the bolt centres

= Dia. of crank pin or big end bearing + 2 × Thickness of bearing liner + Nominal dia. of bolt + Clearance

$= (d_c + 2 \times 3 + d_b + 3)$ mm = 44 + 6 + 12 + 3 = 65

mm ∴ Maximum bending moment acting on the cap,

$$M_C = \frac{F_I \cdot x}{6} = \frac{9490 \cdot 65}{6} = 102\ 810 \text{ N-mm}$$

Section modulus for the cap

$$Z_C = \frac{b_c (t_c)^2}{6} = \frac{58 (t_c)^2}{6} = 9.7 (t_c)^2$$

We know that bending stress (σ_b),

$$80 = \frac{M_C}{Z_C} = \frac{102\ 810}{9.7 (t_c)^2} = \frac{10\ 600}{(t_c)^2}$$

∴ $(t_c)^2 = 10\ 600 / 80 = 132.5$ or $t_c = 11.5$ mm **Ans.**

Let us now check the design for the induced bending stress due to inertia bending forces on the connecting rod (*i.e.* whipping stress).

We know that mass of the connecting rod per metre length,

$$m_1 = \text{Volume} \times \text{density} = \text{Area} \times \text{length} \times \text{density}$$

$$= A \times l \times \rho = 11 t^2 \times l \times \rho \qquad \ldots(\because A = 11 t^2)$$

$$= 11 (0.007)^2 (0.38)\ 8000 = 1.64 \text{ kg}$$

$$\ldots[\because \rho = 8\ 000 \text{ kg/m}^3 \text{ (given)}]$$

∴ Maximum bending moment,

$$M_{max} = m \cdot \omega^2 \cdot r \times \frac{l}{9\sqrt{3}} = m_1 \cdot \omega^2 \cdot r \times \frac{l^2}{9\sqrt{3}} \qquad \ldots(\because m = m_1 \cdot l)$$

$$= 1.64 \left(\frac{2\pi \cdot 1800}{60}\right)^2 (0.095) \frac{(0.38)^2}{9\sqrt{3}} = 51.3 \text{ N-m}$$

$$= 51\ 300 \text{ N-mm}$$

and section modulus, $Z_{xx} = \frac{I_{xx}}{5t/2} = \frac{419\, t^4}{12} \times \frac{2}{5t} = 13.97\, t^3 = 13.97 \times 7^3 = 4792 \text{ mm}^3$

∴ Maximum bending stress (induced) due to inertia bending forces or whipping stress,

$$\sigma_{b(max)} = \frac{M_{max}}{Z_{xx}} = \frac{51\ 300}{4792} = 10.7 \text{ N/mm}^2$$

Since the maximum bending stress induced is less than the allowable bending stress of 80 N/mm^2, therefore the design is safe.

UNIT III DESIGN OF CRANKSHAFT

Balancing of I.C. engines

Engine balance refers to those factors in the design, production, engine tuning, maintenance and the operation of an engine that benefit from being balanced. Major considerations are:

- Balancing of structural and operational elements within an engine
- Longevity and performance
- Power and efficiency
- Performance and weight/size/cost
- Environmental cost and utility
- Noise/vibration and performance

This article is currently limited to structural and operational balance within an engine in general, and balancing of piston engine components in particular.

Piston engine balancing is a complicated subject that covers many areas in the design, production, tuning and operation. The engine considered to be well balanced in a particular usage may produce unacceptable level of vibration in another usage for the difference in driven mass and mounting method, and slight variations in resonant frequencies of the environment and engine parts could be big factors in throwing a smooth operation off balance. In addition to the vast areas that need to be covered and the delicate nature, terminologies commonly used to describe engine balance are often incorrectly understood and/or poorly defined not only in casual discussions but also in many articles in respected publications.

Internal combustion piston engines, by definition, are converter devices to transform energy in intermittent combustion into energy in mechanical motion. A slider-crank mechanismis used in creating a chemical reaction on fuel with air (compression and ignition), and converting the energy into rotation (expansion). The intermittent energy source combined with the nature of this mechanism make the engine naturally vibration-prone. Multi-cylinder configuration and many of the engine design elements are reflections of the effort to reduce vibrations through the act of balancing.

This article is organized in six sections:

- "Items to be balanced"
  lists the balancing elements to establish the basics on the causes of imbalance.
- "Types of vibration"
  lists different kinds of vibration as the effects of imbalance.
- "Primary balance"
  discusses the term "Primary balance".
- "Secondary balance"
  explains what Secondary balance is, and how the confusing terminologies 'Primary' and 'Secondary' came into popular use.
- "Inherent balance"
  goes into engine balance discussions on various multi-cylinder configurations.
- "Steam locomotives"
  is an introduction to the balancing of 2-cylinder locomotives and includes the wheel hammer effect unique to steam locomotives.

There are many factors that can contribute to engine imbalance, and there are many ways to categorize them. The following categories will be used for the purposes of this discussion. In the category descriptions, 'Phase' refers to the timing on the rotation of crankshaft, 'Plane' refers to the location on the crankshaft rotating axis, and 'CG' refers to the center of gravity.

- Mechanical
  - Static Balance - Static balance refers to the balancing of weight and the location of CG on moving parts.

  1. Reciprocating mass - e.g. Piston and connecting rod weight and CG uniformity.

  2. Rotating mass - e.g. Crank web weight uniformity and flywheel eccentricity (or lack there of)

- Dynamic Balance - In order for a mass to start moving from rest or change direction, it needs to be accelerated. A force is required to accelerate a mass. According to Newton's 3rd law of motion, there will be an counter force in the opposite direction of equal size. Dynamic balance refers to the balancing of these forces and forces due to friction.

All accelerations of a mass can be divided into two components in opposite directions. For example, in order for a piston in a single cylinder engine to be accelerated upward, something must receive (support) the downward force, and it is usually the mass of the entire engine that moves downward a bit as there is no counter-moving piston. This means one cause of engine vibration usually appears in two opposing directions. Often the movement or deflection in one direction appears on a moving mass, and the other direction appears on the entire engine, but sometimes both sides appear on moving parts, e.g. a torsional vibration in a crankshaft, or a push-pull cyclic stress in a chain or connecting rod. In other cases, one side is a deflection of a static part, the energy in which is converted into heat and dissipated into the coolant.

- Reciprocating mass - Piston mass needs to be accelerated and decelerated, resisting a smooth rotation of a crankshaft. In addition to the up-down movement of a piston, a connecting rod big end swings left and right and up and down while it rotates. In order to simplify the motion of a crank slider mechanism, the connecting rod/piston assembly is generally divided into two mass groups, a reciprocating mass, and a rotating mass. The big end of the rod is generally said to be rotating while the small end is said to be reciprocating. In truth, however, both ends both reciprocate and rotate.

3. Phase balance - e.g. Pistons on 60 or 90° V6 without an offset crankshaft reciprocate with unevenly spaced phases in a crank rotation

4. Plane balance - e.g. Boxer Twin pistons travel on two different rotational planes of the crankshaft, which creates forces to rock the engine on Z-axis[note 1]

- Rotating mass

5. Phase balance - e.g. Imbalance in camshaft rotating mass can generate a vibration with a frequency equal to once in 2 crank rotations in a 4 cycle engine
6. Plane balance - e.g. Boxer Twin crankshaft without counterweights rocks the engine on Z-axis[1]
7. Torsional balance - If the rigidity of crank throws on an inline 4 cylinder engine is uniform, the crank throw farthest from the clutch surface (usually called cylinder #1) normally shows the biggest torsional deflection. It is usually impossible to make these deflections uniform across multiple cylinders except on a radial engine. SeeTorsional vibration

- **8.** Static mass - A single cylinder 10 HP engine weighing a ton is very smooth, because the forces that comprise its imbalance in operation must move a large mass to create a vibration. As power to weight ratio is important in the design of an engine, the weight of a crankcase, cylinder block, cylinder head, etc. (i.e. static mass) are usually made as light as possible within the limitations of strength, cost and safety margin, and are often excluded in the consideration of engine balance.

However, most vibrations of an engine are small movements of the engine itself, and are thus determined by the engine weight, rigidity, location of CG, and how much its mass is concentrated around the CG. These are crucial factors in engine dynamic balance, which is defined for the whole engine in reciprocal and rotational movements as well as in bending and twisting deflections on the X, Y and Z axis. All of these are important factors in the design of engine mounts and the rigidity of static parts.
It is important to recognize that some moving masses must be considered a part of static mass depending on the kind of dynamic

balance under consideration (e.g. camshaft weight in analyzing the Y-axis[note 1] rotational vibration of an engine).

- Friction

9. Slide resistance balance - A piston slides in a cylinder with friction. A ball in a ball bearing also slides as the diameter of inner and outer laces are different and the distance of circumference differs from the inside and out. When a ball bearing is used as the main bearing on a crankshaft (which is rarely the case), eccentricity of the cage (lace) normally create phase imbalance in slide friction. Friction forces for shell bearings (the most common type of bearings) are dependent upon diameter and width, which determine bearing surface area. This needs to be balanced for the pressure and the rotational speed of the load. Different main bearing sizes on a crankshaft create plane imbalance in slide friction.

10. Rolling resistance balance - e.g. A ball in a ball bearing generates friction while rolling in it's cage

SIGNIFICANCE OF FIRING ORDER

The **firing order** is the sequence of power delivery of each cylinder in a multi-cylinder reciprocating engine.

This is achieved by sparking of the spark plugs in a gasoline engine in the correct order, or by the sequence of fuel injection in a Diesel engine. When designing an engine, choosing an appropriate firing order is critical to minimizing vibration, to improve engine balance and achieving smooth running, for long engine fatigue life and user comfort, and heavily influences crankshaft design.

Cylinder numbering and firing orders for various engine layouts[edit]

In a straight engine the spark plugs (and cylinders) are numbered, starting with #1, usually from the front of the engine to the rear.

1-3-5-2-4 would be the firing order for this 5-cylinder radial engine.

In a radial engine the cylinders are numbered around the circle, with the #1 cylinder at the top. There are always an odd number of cylinders in each bank, as this allows for a constant alternate cylinder firing order: for example, with a single bank of 7 cylinders, the order would be 1-3-5-7-2-4-6. Moreover, unless there is an odd number of cylinders, the ring cam around the nose of the engine would be unable to provide the inlet valve open - exhaust valve open sequence required by the four-stroke cycle.

The cylinder numbering scheme used by some manufacturers on their V engines is based on "folding" the engine into an inline type.

In a V engine, cylinder numbering varies among manufacturers. Generally speaking, the most forward cylinder is numbered 1, but some manufacturers will then continue numbering along that bank first (so that side of the engine would be 1-2-3-4, and the opposite bank would be 5-6-7-8) while others will number the cylinders from front to back along the crankshaft, so one bank would be 1-3-5-7 and the other bank would be 2-4-6-8. (In this example, a V8 is assumed). To further complicate matters, manufacturers may not have used the same system for all of their engines. It is important to check the numbering system used before comparing firing orders, because the order will vary significantly with crankshaft design (see crossplane).

As an example, the Chevrolet Small-Block engine has cylinders 1-3-5-7 on the left hand side of the car, and 2-4-6-8 on the other side, and uses a firing order of 1-8-4-3-6-5-7-2.[1] Note that the order alternates irregularly between the left and right banks; this is what causes the 'burbling' sound of this type of engine.[2]

In most Audi and Ford V8 engines cylinders 1-2-3-4 are on the right hand side of the car, with 5-6-7-8 are on the left.

This means that Chevy Generation 1 Small Block V8 engines and Ford 302 V8s (5.8L, 5.0L, 7.5L) have an identical firing pattern despite having a different firing order.

Likewise, the firing pattern is the same for Chevrolet & Chrysler V8 engines with a firing order of 1-8-4-3-6-5-7-2, and for Ford's V8 engines with a firing order of 1-5-4-2-6-3-7-8.

An exception is the Ford Flathead V8 where the number 1 cylinder is on the right front of the engine (same as other Ford V8's) but this cylinder is not the front cylinder of the engine. In this case number 5 is the front cylinder. A similar situation exists with the Pontiac V8's 455 etc. where the cylinders are numbered like a Chevrolet V8 but the right side bank is in front(like a Ford), this puts cylinder number 2 in front of number 1.

Crankshaft

A crankshaft (*i.e.* a shaft with a crank) is used to convert reciprocating motion of the piston into
rotatory motion or vice versa. The crankshaft consists of the shaft parts which revolve in the main

bearings, the crankpins to which the big ends of the connecting rod are connected, the crank arms or webs (also called cheeks) which connect the crankpins and the shaft parts. The crankshaft, depending upon the position of crank, may be divided into the following two types :

1. Side crankshaft or overhung crankshaft, as shown in Fig. 32.15 (*a*),
2. And centre crankshaft

Design Procedure for Crankshaft

The following procedure may be adopted for designing a crankshaft.

1. First of all, find the magnitude of the various loads on the crankshaft.

2. Determine the distances between the supports and their position with respect to the loads.

3. For the sake of simplicity and also for safety, the shaft is considered to be supported at the centres of the bearings and all the forces and reactions to be acting at these points. The distances between the supports depend on the length of the bearings, which in turn depend on the diameter of the shaft because of the allowable bearing pressures.

4. The thickness of the cheeks or webs is assumed to be from 0.4 ds to 0.6 ds, where ds is the diameter of the shaft. It may also be taken as 0.22D to 0.32 D, where D is the bore of cylinder in mm.

5. Now calculate the distances between the supports.

6. Assuming the allowable bending and shear stresses, determine the main dimensions of the crankshaft.

Notes: 1. The crankshaft must be designed or checked for at least two crank positions. Firstly, when the crankshaft is subjected to maximum bending moment and secondly when the crankshaft is subjected to maximum twisting moment or torque.

2. The additional moment due to weight of flywheel, belt tension and other forces must be considered.

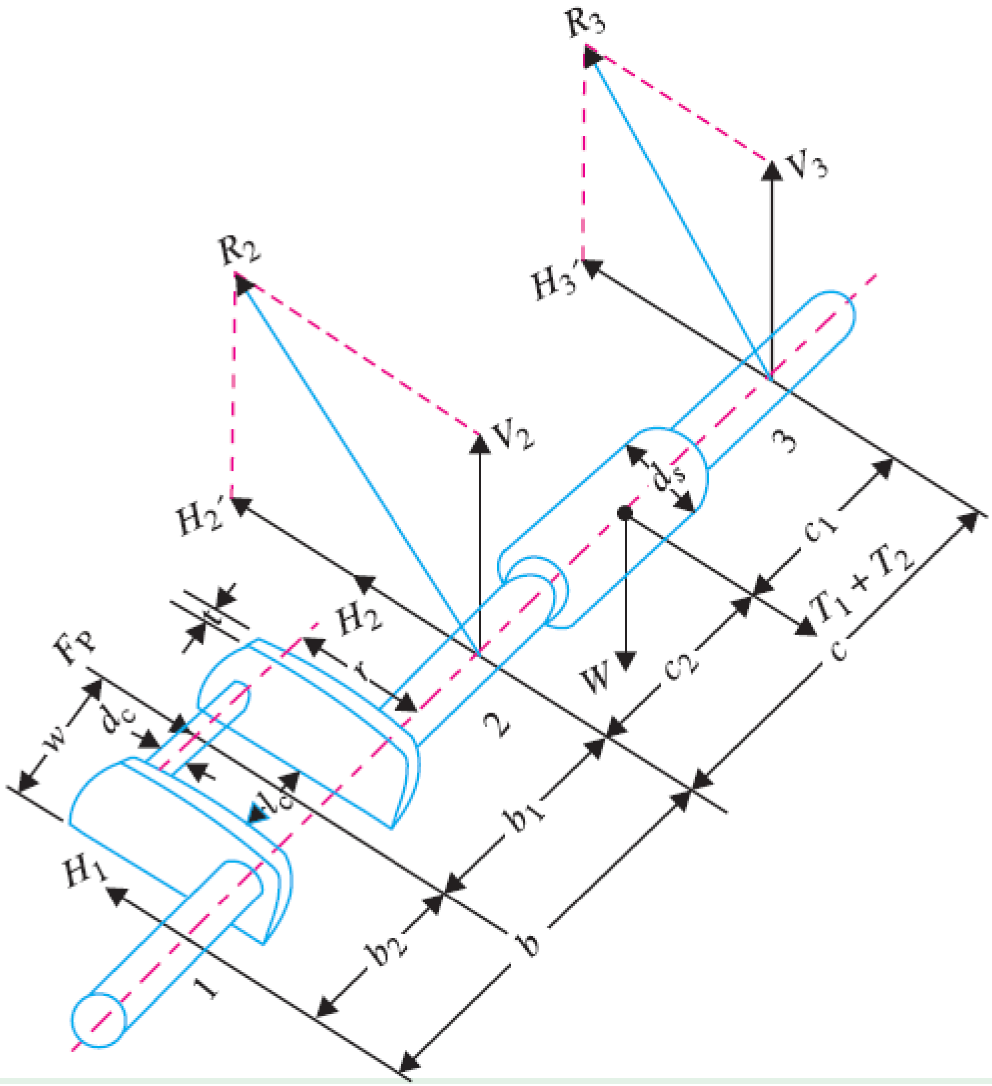
3. It is assumed that the effect of bending moment does not exceed two bearings between which a force is considered.

32.20 Design of Centre Crankshaft

We shall design the centre crankshaft by considering the two crank possitions, *i.e.* when the crank is at dead centre (or when the crankshaft is subjected to maximum bending moment) and when the crank is at angle at which the twisting moment is maximum. These two cases are discussed in detail as below :

1. ***When the crank is at dead centre***. At this position of the crank, the maximum gas pressure on the piston will transmit maximum force on the crankpin in the plane of the crank causing only bending of the shaft. The crankpin as well as ends of the crankshaft will be only subjected to bending moment.

Thus, when the crank is at the dead centre, the bending moment on the shaft is maximum and the twisting moment is zero.


R3
V3
H3′
R2
V2
H2′
H2
ds
3
c1
T1 + T2
c
W
c2
2
Fp
t
r
w
dc
lc
b1
b
H1
b2
1

Let D = Piston diameter or cylinder bore in mm,

p = Maximum intensity of pressure on the piston in N/mm^2,

W = Weight of the flywheel acting downwards in N, and

*$T_1 + T_2$ = Resultant belt tension or pull acting horizontally in N.

The thrust in the connecting rod will be equal to the gas load on the piston (F_P). We know that gas load on the piston,

$$F_P = \frac{\pi}{4} \times D^2 \times p$$

Due to this piston gas load (F_P) acting horizontally, there will be two horizontal reactions H_1 and H_2 at bearings 1 and 2 respectively, such that

$$H_1 = \frac{F_P \times b_1}{b}; \quad \text{and} \quad H_2 = \frac{F_P \times b_2}{b}$$

Due to the weight of the flywheel (W) acting downwards, there will be two vertical reactions V_2 and V_3 at bearings 2 and 3 respectively, such that

$$V_2 = \frac{W \times c_1}{c}; \quad \text{and} \quad V_3 = \frac{W \times c_2}{c}$$

Now due to the resultant belt tension ($T_1 + T_2$), acting horizontally, there will be two horizontal reactions H_2' and H_3' at bearings 2 and 3 respectively, such that

$$H_2' = \frac{(T_1 + T_2)\, c_1}{c}; \quad \text{and} \quad H_3' = \frac{(T_1 + T_2)\, c_2}{c}$$

The resultant force at bearing 2 is given by

$$R_2 = \sqrt{(H_2 + H_2')^2 + (V_2)^2}$$

and the resultant force at bearing 3 is given by

$$R_3 = \sqrt{(H_3)^2 + (V_3)^2}$$

Now the various parts of the centre crankshaft are designed for bending only, as discussed below:

(a) Design of crankpin

Let d_c = Diameter of the crankpin in mm,

l_c = Length of the crankpin in mm,

σ_b = Allowable bending stress for the crankpin in N/mm^2.

We know that bending moment at the centre of the crankpin,

$$M_C = H_1 \cdot b_2 \quad \text{...}(i)$$

We also know that

$$M_C = \frac{\pi}{32} (d_c)^3 \sigma_b \quad \text{...}(ii)$$

From equations (*i*) and (*ii*), diameter of the crankpin is determined. The length of the crankpin is given by

$$l_c = \frac{F_P}{d_c \cdot p_b}$$

where p_b = Permissible bearing pressure in N/mm^2.

(b) Design of left hand crank web

The crank web is designed for eccentric loading. There will be two stresses acting on the crank web, one is direct compressive stress and the other is bending stress due to piston gas load (F_P).

where σ_{bR} = Bending stress in the radial direction, and

$$Z = \text{Section modulus} = \frac{1}{6} \times w \cdot t^2$$

From equations (*i*) and (*ii*), the value of bending stress σ_{bR} is determined.

The bending moment due to the tangential component of F_Q is maximum at the juncture of crank and shaft. It is given by

$$M_T = F_T\left[r - \frac{d_{s1}}{2}\right] \quad \text{... (iii)}$$

where d_{s1} = Shaft diameter at juncture of right hand crank arm, *i.e.* at bearing 2.

We also know that $M_T = \sigma_{bT} \times Z = \sigma_{bT} \times \frac{1}{6} \times t.w^2$ ... (*iv*)

where σ_{bT} = Bending stress in tangential direction.

From equations (*iii*) and (*iv*), the value of bending stress σ_{bT} is determined.

The direct compressive stress is given by,

$$\sigma_d = \frac{F_R}{2\,w.t}$$

The maximum compressive stress (σ_c) will occur at the upper left corner of the cross-section of the crank.

$$\therefore \quad \sigma_c = \sigma_{bR} + \sigma_{bT} + \sigma_d$$

Now, the twisting moment on the arm,

$$T = H_{T1}\left(b_2 + \frac{l_c}{2}\right) - F_T \times \frac{l_c}{2} = H_{T2}\left(b_1 - \frac{l_c}{2}\right)$$

We know that shear stress on the arm,

$$\tau = \frac{T}{Z_P} = \frac{4.5\,T}{w.t^2}$$

where Z_P = Polar section modulus $= \frac{w.t^2}{4.5}$

$\therefore$ Maximum or total combined stress,

$$(\sigma_c)_{max} = \frac{\sigma_c}{2} + \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2}$$

The thickness (t) of the crank web is given empirically as

$$t = 0.4\,d_s \text{ to } 0.6\,d_s$$
$$= 0.22D \text{ to } 0.32D$$
$$= 0.65\,d_c + 6.35 \text{ mm}$$

where d_s = Shaft diameter in mm,
D = Bore diameter in mm, and
d_c = Crankpin diameter in mm,

The width of crank web (w) is taken as

$$w = 1.125\,d_c + 12.7 \text{ mm}$$

We know that maximum bending moment on the crank web,

$$M = H_1\left(b_2 - \frac{l_c}{2} - \frac{t}{2}\right)$$

and section modulus, $Z = \frac{1}{6} \times w.\,t^2$

$\therefore$ Bending stress, $$\sigma_b = \frac{M}{Z} = \frac{6H_1\left(b_2 - \frac{l_c}{2} - \frac{t}{2}\right)}{w.\,t^2}$$

and direct compressive stress on the crank web,

$$\sigma_c = \frac{H_1}{w.\,t}$$

$\therefore$ Total stress on the crank web

$$= \text{Bending stress} + \text{Direct stress} = \sigma_b + \sigma_c$$
$$= \frac{6H_1\left(b_2 - \frac{l_c}{2} - \frac{t}{2}\right)}{w.\,t^2} + \frac{H_1}{w.\,t}$$

This total stress should be less than the permissible bending stress.

(c) Design of right hand crank web

The dimensions of the right hand crank web (*i.e.* thickness and width) are made equal to left hand crank web from the balancing point of view.

(d) Design of shaft under the flywheel

Let d_s = Diameter of shaft in mm.

We know that bending moment due to the weight of flywheel,

$$M_W = V_3 . c_1$$

and bending moment due to belt tension,

$$M_T = H_3' . c_1$$

These two bending moments act at right angles to each other. Therefore, the resultant bending moment at the flywheel location,

$$M_S = \sqrt{(M_W)^2 + (M_T)^2} = \sqrt{(V_3 . c_1)^2 + (H_3 . c_1)^2} \quad \text{...}(i)$$

We also know that the bending moment at the shaft,

$$M_S = \frac{\pi}{32}(d_s)^3\,\sigma_b \quad \text{...}(ii)$$

where σ_b = Allowable bending stress in N/mm².

From equations (*i*) and (*ii*), we may determine the shaft diameter (d_s).

The value of (ʃc)*max* should be within safe limits. If it exceeds the safe value, then the dimension
w may be increased because it does not affect other dimensions.

(*e*) *Design of left hand crank web*

Since the left hand crank web is not stressed to the extent as the right hand crank web, therefore,
the dimensions for the left hand crank web may be made same as for right hand crank web.

(*f*) *Design of crankshaft bearings*

The bearing 2 is the most heavily loaded and should be checked for the safe bearing pressure.
We know that the total reaction at the bearing 2,

$$R_2 = \frac{F_P}{2} + \frac{W}{2} + \frac{T_1 + T_2}{2}$$

$$\therefore \text{ Total bearing pressure } = \frac{R_2}{l_2 \cdot d_{s1}}$$

where l_2 = Length of bearing 2.

Design a plain carbon steel centre crankshaft for a single acting four stroke single cylinder engine for the following data:
Bore = 400 mm ; Stroke = 600 mm ; Engine speed = 200 r.p.m. ; Mean effective pressure = 0.5
N/mm2; Maximum combustion pressure = 2.5 N/mm2; Weight of flywheel used as a pulley = 50 kN;
Total belt pull = 6.5 kN.
When the crank has turned through 35° from the top dead centre, the pressure on the piston is
1N/mm2 and the torque on the crank is maximum. The ratio of the connecting rod length to the crank
radius is 5. Assume any other data required for the design.

Solution. Given : *D* = 400 mm ; *L* = 600 mm or *r* = 300 mm ; *pm* = 0.5 N/mm2 ; *p* = 2.5 N/mm2 ;
W = 50 kN ; *T*1 + *T*2 = 6.5 kN ; ⟨ = 35° ; *p*2 = 1N/mm2 ; *l* / *r* = 5

We shall design the crankshaft for the two positions of the crank, *i.e.* firstly when the crank is at the dead centre ; and secondly when the crank is at an angle of maximum twisting moment.

1. *Design of the crankshaft when the crank is at the dead centre*

We know that the piston gas load,

$$F_P = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4}(400)^2 2.5 = 314\,200\ \text{N} = 314.2\ \text{kN}$$

Assume that the distance (b) between the bearings 1 and 2 is equal to twice the piston diameter (D).

$\therefore \quad b = 2D = 2 \times 400 = 800$ mm

and $\quad b_1 = b_2 = \frac{b}{2} = \frac{800}{2} = 400$ mm

We know that due to the piston gas load, there will be two horizontal reactions H_1 and H_2 at bearings 1 and 2 respectively, such that

$$H_1 = \frac{F_P \times b_1}{b} = \frac{314.2 \times 400}{800} = 157.1\ \text{kN}$$

and

$$H_2 = \frac{F_P \times b_2}{b} = \frac{314.2 \times 400}{800} = 157.1\ \text{kN}$$

Assume that the length of the main bearings to be equal, *i.e.*, $c_1 = c_2 = c/2$. We know that due to the weight of the flywheel acting downwards, there will be two vertical reactions V_2 and V_3 at bearings 2 and 3 respectively, such that

$$V_2 = \frac{W \times c_1}{c} = \frac{W \times c/2}{c} = \frac{W}{2} = \frac{50}{2} = 25\ \text{kN}$$

and

$$V_3 = \frac{W \times c_2}{c} = \frac{W \times c/2}{c} = \frac{W}{2} = \frac{50}{2} = 25\ \text{kN}$$

Due to the resultant belt tension $(T_1 + T_2)$ acting horizontally, there will be two horizontal reactions H_2' and H_3' respectively, such that

$$H_2' = \frac{(T_1 + T_2)\, c_1}{c} = \frac{(T_1 + T_2)\, c/2}{c} = \frac{T_1 + T_2}{2} = \frac{6.5}{2} = 3.25\ \text{kN}$$

and

$$H_3' = \frac{(T_1 + T_2)\, c_2}{c} = \frac{(T_1 + T_2)\, c/2}{c} = \frac{T_1 + T_2}{2} = \frac{6.5}{2} = 3.25\ \text{kN}$$

Now the various parts of the crankshaft are designed as discussed below:

(a) Design of crankpin

Let d_c = Diameter of the crankpin in mm ;

l_c = Length of the crankpin in mm ; and

σ_b = Allowable bending stress for the crankpin. It may be assumed as 75 MPa or N/mm².

We know that the bending moment at the centre of the crankpin,

$$M_C = H_1 \cdot b_2 = 157.1 \times 400 = 62\,840\ \text{kN-mm} \quad \text{...}(i)$$

We also know that

$$M_C = \frac{\pi}{32}(d_c)^3 \sigma_b = \frac{\pi}{32}(d_c)^3 75 = 7.364 (d_c)^3\ \text{N-mm}$$

$$= 7.364 \times 10^{-3} (d_c)^3\ \text{kN-mm} \quad \text{...}(ii)$$

Equating equations (*i*) and (*ii*), we have

$(d_c)^3 = 62\,840 / 7.364 \times 10^{-3} = 8.53 \times 10^6$

or $\quad d_c = 204.35$ say 205 mm **Ans.**

We know that length of the crankpin,

$$l_c = \frac{F_P}{d_c \cdot p_b} = \frac{314.2 \times 10^3}{205 \times 10} = 153.3 \text{ say } 155 \text{ mm} \quad \textbf{Ans.}$$

...(Taking p_b = 10 N/mm²)

(b) Design of left hand crank web

We know that thickness of the crank web,

$t = 0.65\, d_c + 6.35$ mm

$= 0.65 \times 205 + 6.35 = 139.6$ say 140 mm **Ans.**

and width of the crank web, $w = 1.125\, d_c + 12.7$ mm

$$= 1.125 \times 205 + 12.7 = 243.3 \text{ say } 245 \text{ mm } \textbf{Ans.}$$

We know that maximum bending moment on the crank web,

$$M = H_1\left(b_2 - \frac{l_c}{2} - \frac{t}{2}\right)$$

$$= 157.1\left(400 - \frac{155}{2} - \frac{140}{2}\right) = 39\ 668 \text{ kN-mm}$$

Section modulus, $Z = \dfrac{1}{6} \times w.t^2 = \dfrac{1}{6} \times 245\ (140)^2 = 800 \times 10^3 \text{ mm}^3$

$\therefore$ Bending stress, $\sigma_b = \dfrac{M}{Z} = \dfrac{39\ 668}{800 \times 10^3} = 49.6 \times 10^{-3} \text{ kN/mm}^2 = 49.6 \text{ N/mm}^2$

We know that direct compressive stress on the crank web,

$$\sigma_c = \frac{H_1}{w.t} = \frac{157.1}{245 \times 140} = 4.58 \times 10^{-3} \text{ kN/mm}^2 = 4.58 \text{ N/mm}^2$$

$\therefore$ Total stress on the crank web

$$= \sigma_b + \sigma_c = 49.6 + 4.58 = 54.18 \text{ N/mm}^2 \text{ or MPa}$$

Since the total stress on the crank web is less than the allowable bending stress of 75 MPa, therefore, the design of the left hand crank web is safe.

(c) Design of right hand crank web

From the balancing point of view, the dimensions of the right hand crank web (*i.e.* thickness and width) are made equal to the dimensions of the left hand crank web.

(d) Design of shaft under the flywheel

Let d_s = Diameter of the shaft in mm.

Since the lengths of the main bearings are equal, therefore

$$l_1 = l_2 = l_3 = 2\left(\frac{b}{2} - \frac{l_c}{2} - t\right) = 2\left(400 - \frac{155}{2} - 140\right) = 365 \text{ mm}$$

Assuming width of the flywheel as 300 mm, we have

$$c = 365 + 300 = 665 \text{ mm}$$

Allowing space for gearing and clearance, let us take $c = 800$ mm.

$$\therefore \qquad c_1 = c_2 = \frac{c}{2} = \frac{800}{2} = 400 \text{ mm}$$

We know that bending moment due to the weight of flywheel,

$$M_W = V_3 \cdot c_1 = 25 \times 400 = 10\,000 \text{ kN-mm} = 10 \times 10^6 \text{ N-mm}$$

and bending moment due to the belt pull,

$$M_T = H_3' \cdot c_1 = 3.25 \times 400 = 1300 \text{ kN-mm} = 1.3 \times 10^6 \text{ N-mm}$$

$\therefore$ Resultant bending moment on the shaft,

$$M_S = \sqrt{(M_W)^2 + (M_T)^2} = \sqrt{(10 \times 10^6)^2 + (1.3 \times 10^6)^2}$$
$$= 10.08 \times 10^6 \text{ N-mm}$$

We also know that bending moment on the shaft (M_S),

$$10.08 \times 10^6 = \frac{\pi}{32}(d_s)^3 \sigma_b = \frac{\pi}{32}(d_s)^3 42 = 4.12\,(d_s)^3$$

$$\therefore \qquad (d_s)^3 = 10.08 \times 10^6 / 4.12 = 2.45 \times 10^6 \text{ or } d_s = 134.7 \text{ say } 135 \text{ mm}$$ **Ans.**

2. Design of the crankshaft when the crank is at an angle of maximum twisting moment

We know that piston gas load,

$$F_P = \frac{\pi}{4} \times D^2 \times p' = \frac{\pi}{4}(400)^2 1 = 125\,680 \text{ N} = 125.68 \text{ kN}$$

In order to find the thrust in the connecting rod (F_Q), we should first find out the angle of inclination of the connecting rod with the line of stroke (*i.e.* angle ϕ). We know that

$$\sin\phi = \frac{\sin\theta}{l/r} = \frac{\sin 35°}{5} = 0.1147$$

$$\therefore \qquad \phi = \sin^{-1}(0.1147) = 6.58°$$

We know that thrust in the connecting rod,

$$F_Q = \frac{F_P}{\cos\phi} = \frac{125.68}{\cos 6.58°} = \frac{125.68}{0.9934} = 126.5 \text{ kN}$$

Tangential force acting on the crankshaft,

$$F_T = F_Q \sin(\theta + \phi) = 126.5 \sin(35° + 6.58°) = 84 \text{ kN}$$

and radial force, $$F_R = F_Q \cos(\theta + \phi) = 126.5 \cos(35° + 6.58°) = 94.6 \text{ kN}$$

Due to the tangential force (F_T), there will be two reactions at bearings 1 and 2, such that

$$H_{T1} = \frac{F_T \times b_1}{b} = \frac{84 \times 400}{800} = 42 \text{ kN}$$

and $$H_{T2} = \frac{F_T \times b_2}{b} = \frac{84 \times 400}{800} = 42 \text{ kN}$$

Due to the radial force (F_R), there will be two reactions at bearings 1 and 2, such that

$$H_{R1} = \frac{F_R \times b_1}{b} = \frac{94.6 \times 400}{800} = 47.3 \text{ kN}$$

$$H_{R2} = \frac{F_R \times b_2}{b} = \frac{94.6 \times 400}{800} = 47.3 \text{ kN}$$

Now the various parts of the crankshaft are designed as discussed below:

(a) Design of crankpin

Let d_c = Diameter of crankpin in mm.

We know that the bending moment at the centre of the crankpin,

$$M_C = H_{R1} \times b_2 = 47.3 \times 400 = 18\ 920 \text{ kN-mm}$$

and twisting moment on the crankpin,

$$T_C = H_{T1} \times r = 42 \times 300 = 12\ 600 \text{ kN-mm}$$

∴ Equivalent twisting moment on the crankpin,

$$T_e = \sqrt{(M_C)^2 + (T_C)^2} = \sqrt{(18\ 920)^2 + (12\ 600)^2}$$
$$= 22\ 740 \text{ kN-mm} = 22.74 \times 10^6 \text{ N-mm}$$

We know that equivalent twisting moment (T_e),

$$22.74 \times 10^6 = \frac{\pi}{16}(d_c)^3 \tau = \frac{\pi}{16}(d_c)^3 35 = 6.873\ (d_c)^3$$

...(Taking τ = 35 MPa or N/mm²)

∴ $(d_c)^3 = 22.74 \times 10^6 / 6.873 = 3.3 \times 10^6$ or $d_c = 149$ mm

Since this value of crankpin diameter (*i.e.* d_c = 149 mm) is less than the already calculated value of d_c = 205 mm, therefore, we shall take d_c = 205 mm. **Ans.**

(b) Design of shaft under the flywheel

Let d_s = Diameter of the shaft in mm.

The resulting bending moment on the shaft will be same as calculated eariler, *i.e.*

$$M_S = 10.08 \times 10^6 \text{ N-mm}$$

and twisting moment on the shaft,

$$T_S = F_T \times r = 84 \times 300 = 25\ 200 \text{ kN-mm} = 25.2 \times 10^6 \text{ N-mm}$$

∴ Equivalent twisting moment on shaft,

$$T_e = \sqrt{(M_S)^2 + (T_S)^2}$$
$$= \sqrt{(10.08 \times 10^6)^2 + (25.2 \times 10^6)^2} = 27.14 \times 10^6 \text{ N-mm}$$

We know that equivalent twisting moment (T_e),

$$27.14 \times 10^6 = \frac{\pi}{16}(d_s)^3 \tau = \frac{\pi}{16}(135)^3 \tau = 483\ 156\ \tau$$

∴ $\tau = 27.14 \times 10^6 / 483\ 156 = 56.17$ N/mm²

From above, we see that by taking the already calculated value of *ds* = 135 mm, the induced shear stress is more than the allowable shear stress of 31 to 42 MPa. Hence, the value of d_s is calculated by taking τ = 35 MPa or N/mm² in the above equation, *i.e.*

$$27.14 \times 10^6 = \frac{\pi}{16}(d_s)^3 35 = 6.873\ (d_s)^3$$

∴ $(d_s)^3 = 27.14 \times 10^6 / 6.873 = 3.95 \times 10^6$ or d_s = 158 say 160 mm **Ans.**

(c) Design of shaft at the juncture of right hand crank arm

Let d_{s1} = Diameter of the shaft at the juncture of the right hand crank arm.

We know that the resultant force at the bearing 1,

$$R_1 = \sqrt{(H_{T1})^2 + (H_{R1})^2} = \sqrt{(42)^2 + (47.3)^2} = 63.3 \text{ kN}$$

∴ Bending moment at the juncture of the right hand crank arm,

$$M_{S1} = R_1\left(b_2 + \frac{l_c}{2} + \frac{t}{2}\right) - F_Q\left(\frac{l_c}{2} + \frac{t}{2}\right)$$

$$= 63.3\left(400 + \frac{155}{2} + \frac{140}{2}\right) - 126.5\left(\frac{155}{2} + \frac{140}{2}\right)$$

$$= 34.7 \times 10^3 - 18.7 \times 10^3 = 16 \times 10^3 \text{ kN-mm} = 16 \times 10^6 \text{ N-mm}$$

and twisting moment at the juncture of the right hand crank arm,

$$T_{S1} = F_T \times r = 84 \times 300 = 25\,200 \text{ kN-mm} = 25.2 \times 10^6 \text{ N-mm}$$

∴ Equivalent twisting moment at the juncture of the right hand crank arm,

$$T_e = \sqrt{(M_{S1})^2 + (T_{S1})^2}$$

$$= \sqrt{(16 \times 10^6)^2 + (25.2 \times 10^6)^2} = 29.85 \times 10^6 \text{ N-mm}$$

We know that equivalent twisting moment (T_e),

$$29.85 \times 10^6 = \frac{\pi}{16}(d_{s1})^3 \tau = \frac{\pi}{16}(d_{s1})^3 42 = 8.25\ (d_{s1})^3$$

...(Taking τ = 42 MPa or N/mm²)

∴ $(d_{s1})^3 = 29.85 \times 10^6 / 8.25 = 3.62 \times 10^6$ or $d_{s1} = 153.5$ say 155 mm **Ans.**

(d) Design of right hand crank web

Let σ_{bR} = Bending stress in the radial direction ; and

σ_{bT} = Bending stress in the tangential direction.

We also know that bending moment due to the radial component of F_Q,

$$M_R = H_{R2}\left(b_1 - \frac{l_c}{2} - \frac{t}{2}\right) = 47.3\left(400 - \frac{155}{2} - \frac{140}{2}\right) \text{kN-mm}$$

$$= 11.94 \times 10^3 \text{ kN-mm} = 11.94 \times 10^6 \text{ N-mm} \quad ...(i)$$

We also know that bending moment,

$$M_R = \sigma_{bR} \times Z = \sigma_{bR} \times \frac{1}{6} \times w.t^2 \quad ... \left(\because\ Z = \frac{1}{6} \times w.t^2\right)$$

$$11.94 \times 10^6 = \sigma_{bR} \times \frac{1}{6} \times 245\ (140)^2 = 800 \times 10^3 \sigma_{bR}$$

∴ $\sigma_{bR} = 11.94 \times 10^6 / 800 \times 10^3 = 14.9$ N/mm² or MPa

We know that bending moment due to the tangential component of F_Q,

$$M_T = F_T\left(r - \frac{d_{s1}}{2}\right) = 84\left(300 - \frac{155}{2}\right) = 18\,690 \text{ kN-mm}$$

$$= 18.69 \times 10^6 \text{ N-mm}$$

We also know that bending moment,

$$M_T = \sigma_{bT} \times Z = \sigma_{bT} \times \frac{1}{6} \times t.w^2 \quad ... \left(\because\ Z = \frac{1}{6} \times t.w^2\right)$$

$$18.69 \times 10^6 = \sigma_{bT} \times \frac{1}{6} \times 140\,(245)^2 = 1.4 \times 10^6\ \sigma_{bT}$$

∴ $\sigma_{bT} = 18.69 \times 10^6 / 1.4 \times 10^6 = 13.35$ N/mm² or MPa

Direct compressive stress,

$$\sigma_b = \frac{F_R}{2w \cdot t} = \frac{94.6}{2 \times 245 \times 140} = 1.38 \times 10^{-3} \text{ kN/mm}^2 = 1.38 \text{ N/mm}^2$$

and total compressive stress,

$$\sigma_c = \sigma_{bR} + \sigma_{bT} + \sigma_d$$
$$= 14.9 + 13.35 + 1.38 = 29.63 \text{ N/mm}^2 \text{ or MPa}$$

We know that twisting moment on the arm,

$$T = H_{T2}\left(b_1 - \frac{l_c}{2}\right) = 42\left(400 - \frac{155}{2}\right) = 13\ 545 \text{ kN-mm}$$
$$= 13.545 \times 10^6 \text{ N-mm}$$

and shear stress on the arm,

$$\tau = \frac{T}{Z_P} = \frac{4.5T}{w.t^2} = \frac{4.5 \times 13.545 \times 10^6}{245\ (140)^2} = 12.7 \text{ N/mm}^2 \text{ or MPa}$$

We know that total or maximum combined stress,

$$(\sigma_c)_{max} = \frac{\sigma_c}{2} + \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2}$$
$$= \frac{29.63}{2} + \frac{1}{2}\sqrt{(29.63)^2 + 4\ (12.7)^2} = 14.815 + 19.5 = 34.315 \text{ MPa}$$

Since the maximum combined stress is within the safe limits, therefore, the dimension $w = 245$ mm is accepted.

(e) Design of left hand crank web

The dimensions for the left hand crank web may be made same as for right hand crank web.

(f) Design of crankshaft bearings

Since the bearing 2 is the most heavily loaded, therefore, only this bearing should be checked for bearing pressure.

We know that the total reaction at bearing 2,

$$R_2 = \frac{F_P}{2} + \frac{W}{2} + \frac{T_1 + T_2}{2} = \frac{314.2}{2} + \frac{50}{2} + \frac{6.5}{2} = 185.35 \text{ kN} = 185\ 350 \text{ N}$$

∴ Total bearing pressure

$$= \frac{R_2}{l_2 \cdot d_{s1}} = \frac{185\ 350}{365 \times 155} = 3.276 \text{ N/mm}^2$$

Since this bearing pressure is less than the safe limit of 5 to 8 N/mm^2, therefore, the design is safe.

UNIT IV
DESIGN OF FLYWHEELS

Introduction

A flywheel used in machines serves as a reservoir which stores energy during the period when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than supply. In case of steam engines, internal combustion engines, reciprocating compressors and pumps, the energy is developed during one stroke and the engine is to run for the whole cycle on the energy produced during this one stroke. For example, in I.C. engines, the energy is developed only during power stroke which is much more than the engine load, and no energy is being developed during suction, compression and exhaust strokes in case of four stroke engines and during compression in case of two stroke engines. The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crankshaft during other strokes in which no energy is developed, thus rotating the crankshaft at a uniform speed. A little consideration will show that when the flywheel absorbs energy, its speed increases and when it releases, the speed decreases. Hence a flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed. In machines where the operation is intermittent like punching machines, shearing machines, riveting machines, crushers etc., the flywheel stores energy from the power source during the greater portion of the operating cycle and gives it up during a small period of the cycle. Thus the energy from the power source to the machines is supplied practically at a constant rate throughout the operation.

Coefficient of Fluctuation of Speed

The difference between the maximum and minimum speeds during a cycle is called the ***maximum fluctuation of speed.*** The ratio of the maximum fluctuation of speed to the mean speed is called ***coefficient of fluctuation of speed.***

Let N_1 = Maximum speed in r.p.m. during the cycle,

N_2 = Minimum speed in r.p.m. during the cycle, and

N = Mean speed in r.p.m. = $\dfrac{N_1 + N_2}{2}$

$\therefore$ Coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \quad \text{...(In terms of angular speeds)}$$

$$= \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \quad \text{...(In terms of linear speeds)}$$

The coefficient of fluctuation of speed is a limiting factor in the design of flywheel. It varies depending upon the nature of service to which the flywheel is employed. Table 22.1 shows the permissible values for coefficient of fluctuation of speed for some machines.

Note: The reciprocal of coefficient of fluctuation of speed is known as **coefficient of steadiness** and it is denoted by m.

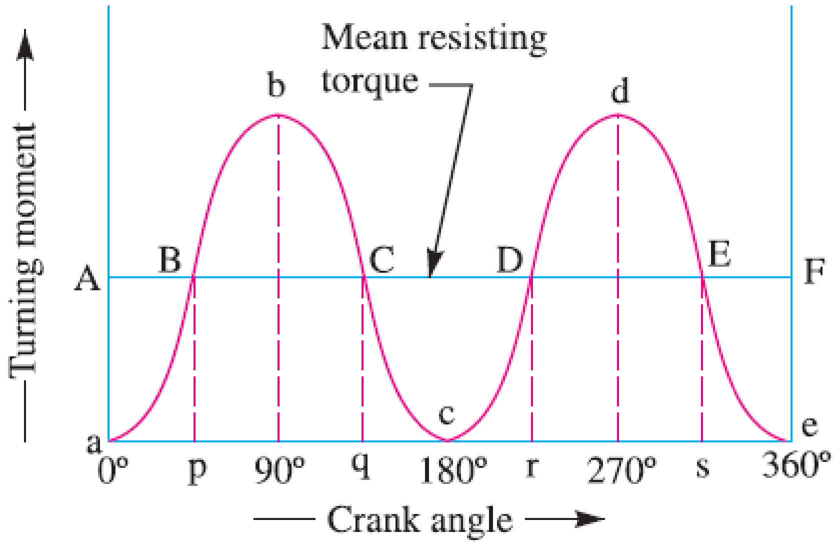
$$\therefore \quad m = \frac{1}{C_S} = \frac{N}{N_1 - N_2} = \frac{\omega}{\omega_1 - \omega_2} = \frac{v}{v_1 - v_2}$$

Fluctuation of Energy

The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. Consider a turning moment diagram for a single cylinder double acting steam engine. The vertical ordinate represents the turning moment and the horizontal ordinate (abscissa) represents the crank angle.

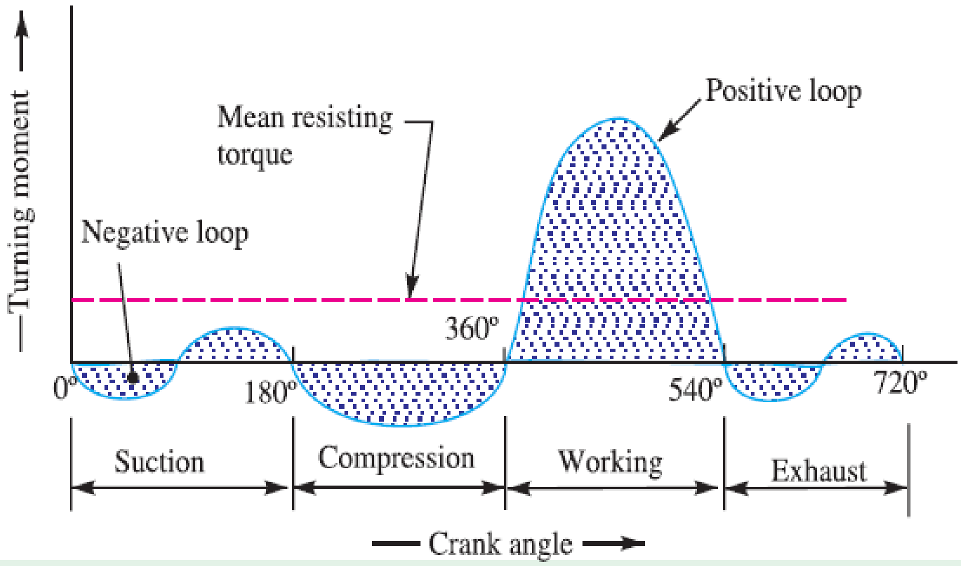
A little consideration will show that the turning moment is zero when the crank angle is zero. It rises to a maximum value when crank angle reaches 90º and it is again zero when crank angle is 180º. This is shown by the curve *abc* in Fig and it represents the turning moment diagram for outstroke.

The curve *cde* is the turning moment diagram for instroke and is somewhat similar to the curve *abc*. Since the work done is the product of the turning moment and the angle turned, therefore the area of the turning moment diagram represents the work done per revolution. In actual practice, the engine is assumed to work against the mean resisting torque, as shown by a horizontal line *AF*. The height of the ordinate *aA* represents the mean height of the turning moment diagram. Since it is assumed that the work done by the turning moment per revolution is equal to the work done against the mean resisting torque, therefore the area of the rectangle *aA Fe* is proportional to the work done against the mean resisting torque.



We see in Fig, that the mean resisting torque line *AF* cuts the turning moment diagram at points *B*, *C*, *D* and *E*. When the crank moves from '*a*' to '*p*' the work done by the engine is equal to the area *aBp*, whereas the energy required is represented by the area *aABp*. In other words, the engine has done less work (equal to the area *aAB*) than the requirement. This amount of energy is taken from the flywheel and hence the speed of the flywheel decreases. Now the crank moves from *p* to *q*, the work done by the engine is equal to the area *pBbCq*, whereas the requirement of energy is represented by the area *pBCq*. Therefore the engine has done more work than the requirement. This excess work (equal to the area *BbC*) is stored in the flywheel and hence the speed of the flywheel increases while the crank moves from *p* to *q*.

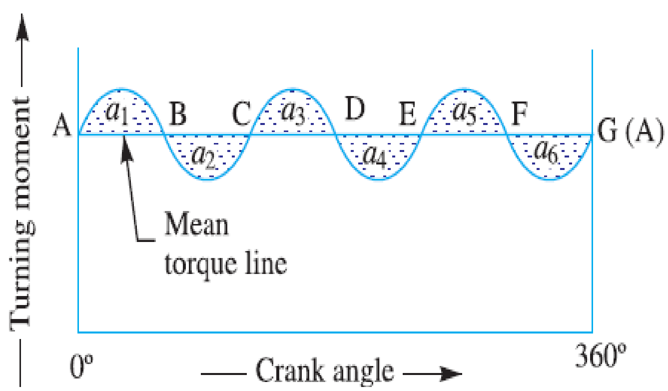
Similarly when the crank moves from *q* to *r*, more work is taken from the engine than is developed. This loss of work is represented by the area *CcD*. To supply this loss, the flywheel gives up some of its energy and thus the speed decreases while the crank moves from *q* to *r*. As the crank moves from *r* to *s*, excess energy is again developed given by the area *DdE* and the speed again increases. As the piston moves from *s* to *e*, again there is a loss of work and the speed decreases. The variations of energy above and below the mean resisting torque line are called ***fluctuation of energy.*** The areas *BbC*, *CcD*, *DdE* etc. represent fluctuations of energy.



Maximum Fluctuation of Energy

A turning moment diagram for a multi-cylinder engine is shown by a wavy curve in Fig.

The horizontal line *AG* represents the mean torque line. Let *a*1, *a*3, *a*5 be the areas above the mean torque line and *a*2, *a*4 and *a*6 be the areas below the mean torque line. These areas represent some quantity of energy which is either added or subtracted from the energy of the moving parts of the engine.



Let the energy in the flywheel at $A = E$, then from Fig. 22.4, we have

Energy at $B = E + a_1$

Energy at $C = E + a_1 - a_2$

Energy at $D = E + a_1 - a_2 + a_3$

Energy at $E = E + a_1 - a_2 + a_3 - a_4$

Energy at $F = E + a_1 - a_2 + a_3 - a_4 + a_5$

Energy at $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6 =$ Energy at A

Let us now suppose that the maximum of these energies is at B and minimum at E.

$\therefore$ Maximum energy in the flywheel

$$= E + a_1$$

and minimum energy in the flywheel

$$= E + a_1 - a_2 + a_3 - a_4$$

$\therefore$ Maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$
$$= (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$$

Coefficient of Fluctuation of Energy

It is defined as the ratio of the maximum fluctuation of energy to the work done per cycle. It is usually denoted by *C*E. Mathematically, coefficient of fluctuation of energy

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

The workdone per cycle may be obtained by using the following relations:

1. Workdone / cycle $= T_{mean} \times \theta$

where T_{mean} = Mean torque, and

θ = Angle turned in radians per revolution

= 2π, in case of steam engines and two stroke internal combustion engines.

= 4π, in case of four stroke internal combustion engines.

The mean torque (T_{mean}) in N-m may be obtained by using the following relation *i.e.*

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

where P = Power transmitted in watts,

N = Speed in r.p.m., and

ω = Angular speed in rad/s = $2\pi N / 60$

2. The workdone per cycle may also be obtained by using the following relation:

$$\text{Workdone / cycle} = \frac{P \times 60}{n}$$

where n = Number of working strokes per minute.

= N, in case of steam engines and two stroke internal combustion engines.

= $N/2$, in case of four stroke internal combustion engines.

The following table shows the values of coefficient of fluctuation of energy for steam engines and internal combustion engines.

Energy Stored in a Flywheel

A flywheel is shown in Fig. We have already discussed that when a flywheel absorbs energy its speed increases and when it gives up energy its speed decreases.

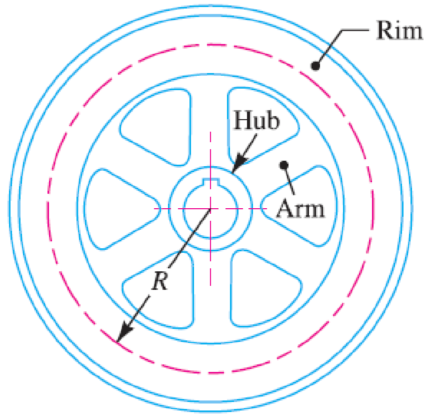


Fig. 22.5. Flywheel.

Let m = Mass of the flywheel in kg,

k = Radius of gyration of the flywheel in metres,

I = Mass moment of inertia of the flywheel about the axis of rotation in kg-m^2 $= m.k^2$,

N_1 and N_2 = Maximum and minimum speeds during the cycle in r.p.m.,

ω_1 and ω_2 = Maximum and minimum angular speeds during the cycle in rad / s,

N = Mean speed during the cycle in r.p.m. $= \dfrac{N_1 + N_2}{2}$,

ω = Mean angular speed during the cycle in rad / s $= \dfrac{\omega_1 + \omega_2}{2}$

C_S = Coefficient of fluctuation of speed $= \dfrac{N_1 - N_2}{N}$ or $\dfrac{\omega_1 - \omega_2}{\omega}$

We know that mean kinetic energy of the flywheel,

$$E = \frac{1}{2} \times I.\omega^2 = \frac{1}{2} \times m.k^2.\omega^2 \text{ (in N-m or joules)}$$

As the speed of the flywheel changes from ω_1 to ω_2, the maximum fluctuation of energy,

$$\Delta E = \text{Maximum K.E.} - \text{Minimum K.E.} = \frac{1}{2} \times I(\omega_1)^2 - \frac{1}{2} \times I(\omega_2)^2$$

$$= \frac{1}{2} \times I \left[(\omega_1)^2 - (\omega_2)^2\right] = \frac{1}{2} \times I\,(\omega_1 + \omega_2)\,(\omega_1 - \omega_2)$$

$$= I.\omega\,(\omega_1 - \omega_2) \qquad \ldots\left(\because \omega = \frac{\omega_1 + \omega_2}{2}\right) \quad \ldots(i)$$

$$= I.\omega^2 \left(\frac{\omega_1 - \omega_2}{\omega}\right) \qquad \ldots[\text{Multiplying and dividing by } \omega]$$

$$= I.\omega^2.C_S = m.k^2.\omega^2.C_S \qquad \ldots(\because I = m.k^2) \quad \ldots(ii)$$

$$= 2\,E.C_S \qquad \ldots\left(\because E = \frac{1}{2} \times I.\omega^2\right) \quad \ldots(iii)$$

The radius of gyration (k) may be taken equal to the mean radius of the rim (R), because the thickness of rim is very small as compared to the diameter of rim. Therefore substituting $k = R$ in equation (*ii*), we have

$$\Delta E = m.R^2.\omega^2.C_S = m.v^2.C_S \qquad \ldots(\because v = \omega.R)$$

From this expression, the mass of the flywheel rim may be determined.

The turning moment diagram for a petrol engine is drawn to the following scales:

Turning moment, 1 mm = 5 N-m; Crank angle, 1 mm = 1°.

The turning moment diagram repeats itself at every half revolution of the engine and the areas above and below the mean turning moment line, taken in order are 295, 685, 40, 340, 960, 270 mm^2.

Determine the mass of 300 mm diameter flywheel rim when the coefficient of fluctuation of speed is 0.3% and the engine runs at 1800 r.p.m. Also determine the cross-section of the rim when the width of the rim is twice of thickness. Assume density of rim material as 7250 kg / m^3.

Solution. Given : D = 300 mm or R = 150 mm = 0.15 m ; C_S = 0.3% = 0.003 ; N = 1800 r.p.m. or $\omega = 2\pi \times 1800 / 60 = 188.5$ rad/s ; ρ = 7250 kg / m^3

Mass of the flywheel

Let m = Mass of the flywheel in kg.

First of all, let us find the maximum fluctuation of energy. The turning moment diagram is shown in Fig. 22.6.

Since the scale of turning moment is 1 mm = 5 N-m, and scale of the crank angle is 1 mm = 1° = π / 180 rad, therefore 1 mm^2 on the turning moment diagram.

$$= 5 \times \pi / 180 = 0.087 \text{ N-m}$$

Let the total energy at $A = E$. Therefore from Fig. 22.6, we find that

Energy at $B = E + 295$

Energy at $C = E + 295 - 685 = E - 390$

Energy at $D = E - 390 + 40 = E - 350$

Energy at $E = E - 350 - 340 = E - 690$

Energy at $F = E - 690 + 960 = E + 270$

Energy at $G = E + 270 - 270 = E$ = Energy at A

From above we see that the energy is maximum at B and minimum at E.

$\therefore$ Maximum energy $= E + 295$

and minimum energy $= E - 690$

We know that maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$
$$= (E + 295) - (E - 690) = 985 \text{ mm}^2$$
$$= 985 \times 0.087 = 86 \text{ N-m}$$

We also know that maximum fluctuation of energy (ΔE),

$$86 = m.R^2.\omega^2.C_S = m\,(0.15)^2\,(188.5)^2\,(0.003) = 2.4\,m$$

$\therefore$ $$m = 86 / 2.4 = 35.8 \text{ kg } \textbf{Ans.}$$

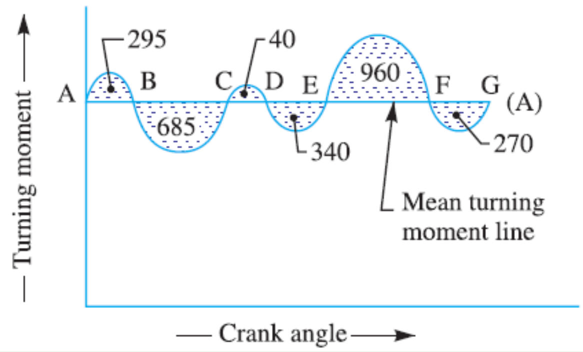


Fig. 22.6

Cross-section of the flywheel rim

Let t = Thickness of rim in metres, and

b = Width of rim in metres = $2\,t$ ...(Given)

$\therefore$ Cross-sectional area of rim,

$$A = b \times t = 2\,t \times t = 2\,t^2$$

We know that mass of the flywheel rim (m),

$$35.8 = A \times 2\pi R \times \rho = 2t^2 \times 2\pi \times 0.15 \times 7250 = 13\,668\,t^2$$

$\therefore$ $t^2 = 35.8 / 13\,668 = 0.0026$ or $t = 0.051 \text{ m} = 51 \text{ mm}$ **Ans.**

and $b = 2\,t = 2 \times 51 = 102 \text{ mm}$ **Ans.**

The intercepted areas between the output torque curve and the mean resistance line of a turning moment diagram for a multicylinder engine, taken in order from one end are as follows:

– 35, + 410, – 285, + 325, – 335, + 260, – 365, + 285, – 260 mm^2.

The diagram has been drawn to a scale of 1 mm = 70 N-m and 1 mm = 4.5°. The engine speed is 900 r.p.m. and the fluctuation in speed is not to exceed 2% of the mean speed.

Find the mass and cross-section of the flywheel rim having 650 mm mean diameter. The density of the material of the flywheel may be taken as 7200 kg / m^3. The rim is rectangular with the width 2 times the thickness. Neglect effect of arms, etc.

Solution. Given : N = 900 r.p.m. or $\omega = 2\pi \times 900 / 60 = 94.26$ rad/s ; $\omega_1 - \omega_2 = 2\%\ \omega$ or

$\dfrac{\omega_1 - \omega_2}{\omega} = C_S = 2\% = 0.02$; D = 650 mm or R = 325 mm = 0.325 m ; ρ = 7200 kg / m^3

Mass of the flywheel rim

Let m = Mass of the flywheel rim in kg.

First of all, let us find the maximum fluctuation of energy. The turning moment diagram for a multi-cylinder engine is shown in Fig. 22.7.

Since the scale of turning moment is 1 mm = 70 N-m and scale of the crank angle is 1 mm = 4.5° = π / 40 rad, therefore 1 mm^2 on the turning moment diagram.

$$= 70 \times \pi / 40 = 5.5 \text{ N-m}$$

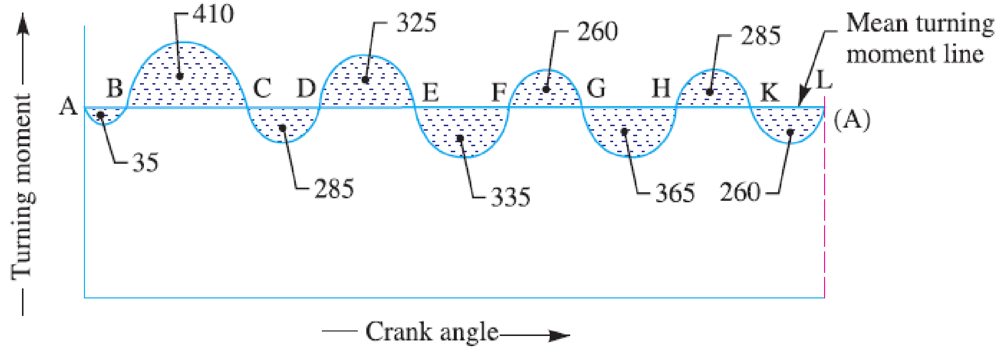


Fig. 22.7

Let the total energy at $A = E$. Therefore from Fig. 22.7, we find that

$$\text{Energy at } B = E - 35$$

$$\text{Energy at } C = E - 35 + 410 = E + 375$$

$$\text{Energy at } D = E + 375 - 285 = E + 90$$

$$\text{Energy at } E = E + 90 + 325 = E + 415$$

$$\text{Energy at } F = E + 415 - 335 = E + 80$$

$$\text{Energy at } G = E + 80 + 260 = E + 340$$

$$\text{Energy at } H = E + 340 - 365 = E - 25$$

$$\text{Energy at } K = E - 25 + 285 = E + 260$$

$$\text{Energy at } L = E + 260 - 260 = E = \text{Energy at } A$$

From above, we see that the energy is maximum at E and minimum at B.

$\therefore$ Maximum energy $= E + 415$

and minimum energy $= E - 35$

We know that maximum fluctuation of energy,

$$= (E + 415) - (E - 35) = 450 \text{ mm}^2$$

$$= 450 \times 5.5 = 2475 \text{ N-m}$$

We also know that maximum fluctuation of energy (ΔE),

$$2475 = \text{m}.R^2.\omega^2.C_S = m\,(0.325)^2\,(94.26)^2\,0.02 = 18.77\,m$$

$$\therefore \quad m = 2475 / 18.77 = 132 \text{ kg } \textbf{Ans.}$$

Cross-section of the flywheel rim

Let t = Thickness of the rim in metres, and

b = Width of the rim in metres = $2\,t$ ...(Given)

$\therefore$ Area of cross-section of the rim,

$$A = b \times t = 2\,t \times t = 2\,t^2$$

We know that mass of the flywheel rim (m),

$$132 = A \times 2\,\pi\,R \times \rho = 2\,t^2 \times 2\,\pi \times 0.325 \times 7200 = 29\,409\,t^2$$

$$\therefore \quad t^2 = 132 / 29\,409 = 0.0044 \quad \text{or} \quad t = 0.067 \text{ m} = 67 \text{ mm } \textbf{Ans.}$$

and $b = 2t = 2 \times 67 = 134$ mm **Ans.**

$-35, +410, -285, +325, -335, +260, -365, +285, -260$ mm^2.

The diagram has been drawn to a scale of 1 mm = 70 N-m and 1 mm = 4.5°. The engine speed is 900 r.p.m. and the fluctuation in speed is not to exceed 2% of the mean speed.

Find the mass and cross-section of the flywheel rim having 650 mm mean diameter. The density of the material of the flywheel may be taken as 7200 kg / m^3. The rim is rectangular with the width 2 times the thickness. Neglect effect of arms, etc.

Solution. Given : $N = 900$ r.p.m. or $\omega = 2\pi \times 900 / 60 = 94.26$ rad/s ; $\omega_1 - \omega_2 = 2\%\ \omega$ or

$\frac{\omega_1 - \omega_2}{\omega} = C_S = 2\% = 0.02$; $D = 650$ mm or $R = 325$ mm $= 0.325$ m ; $\rho = 7200$ kg / m^3

Mass of the flywheel rim

Let m = Mass of the flywheel rim in kg.

First of all, let us find the maximum fluctuation of energy. The turning moment diagram for a multi-cylinder engine is shown in Fig. 22.7.

Since the scale of turning moment is 1 mm = 70 N-m and scale of the crank angle is 1 mm = 4.5° = π / 40 rad, therefore 1 mm^2 on the turning moment diagram.

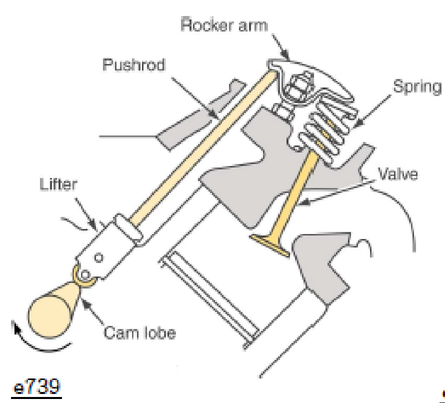
$$= 70 \times \pi / 40 = 5.5 \text{ N-m}$$

UNIT V
DESIGN OF VALVES AND VALVE TRAIN

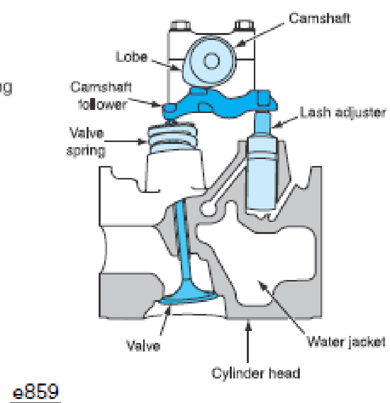
Design aspects of intake & exhaust manifolds

The purpose of the exhaust and inlet processes is to remove the burned gases at the end of the power stroke and admit fresh charge for the next cycle. Indicated power of an ICE at a given speed is proportional to the mass flow rate of air. Inducting the maximum air mass and retaining the mass within the cylinder is the primary goal of the gas exchange processes in engines. Engine gas exchange processes are characterized by volumetric efficiency and it depends on the design of engine subsystems such as manifolds, valves, and ports, as well as engine operating conditions. Supercharging and turbo-charging are used to increase air flow into engine cylinder, and hence power density

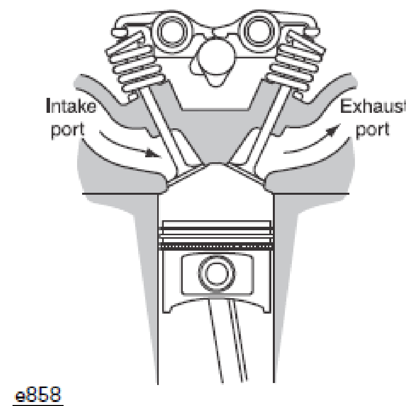
In SI engine, the intake system typically consists of an air filter, a carburettor and throttle or fuel injector and throttle or throttle with individual fuel injectors in each intake port and intake manifold. During the induction process, pressure losses occur as the mixture passes through or by each of these components. The pressure drop depends on engine speed, the flow resistance of the elements in the system, the cross-sectional area through which the fresh charge moves, and the charge density. In a CI engine intake system, the carburettor or EFI system and the throttle plate are absent. The exhaust system typically consists of an exhaust manifold, exhaust pipe, often a catalytic converter for emission control, and a muffler or silencer.



(1)



(2)



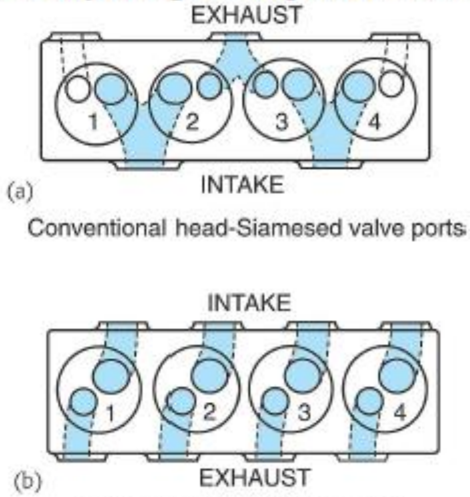
(3)

1. Push-rod assembly (OHV)
2. Single rocker-arm assembly actuated by overhead cam (OHV/OHC)
3. Twin rocker-arm assembly actuated by overhead cam (OHV/OHC)

- **OHV**: Overhead valve
- **OHC**: Overhead cam

Intake & Exhaust Manifolds

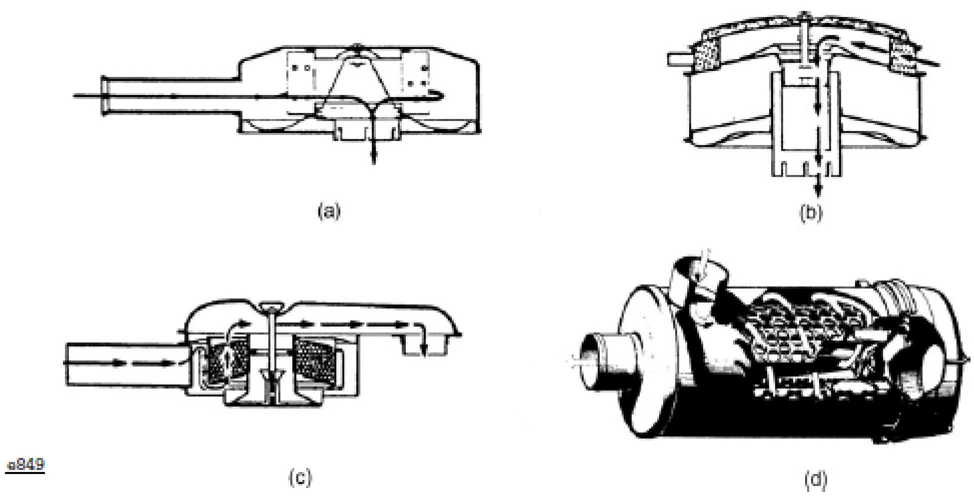
Engine breathing system includes intake & exhaust manifolds that are carefully designed to provide a uniform flow to & from all cylinders.



Alternate head-individual ports

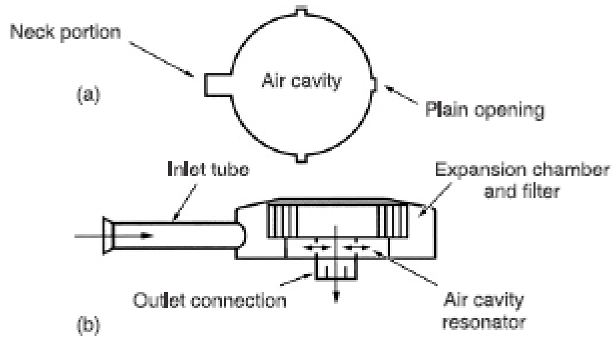
Air Cleaning

Modern air cleaners incorporate at least one the following physical methods of filtration: sieve, impingement and separation.



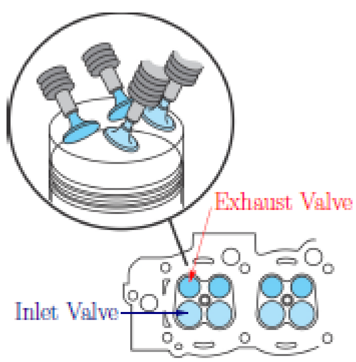
Types of air cleaner: (a) fibre element (b) oil-wetted mesh (c) oil bath and mesh (d) cyclone and fibre element

Air Silencing



e857

The basic structure of an air cleaner/silencer is analogous to that of a Helmholtz resonator, because the air flowing through the main expansion chamber and filter also communicates with an annular air cavity in which there is a purely oscillating movement of air. If suitable length of inlet tube is then matched to the large volume of the main chamber, the air cleaner acting as a Helmholtz resonator can be tuned to respond to an unwanted peak of induction noise.



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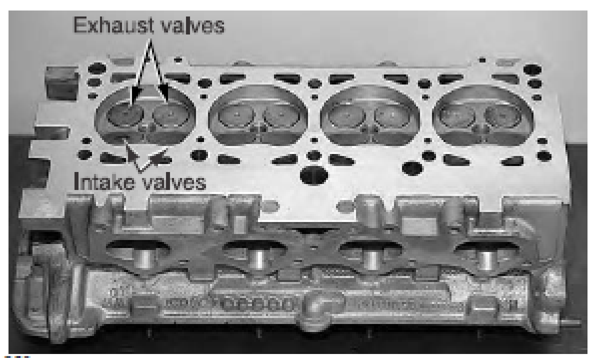
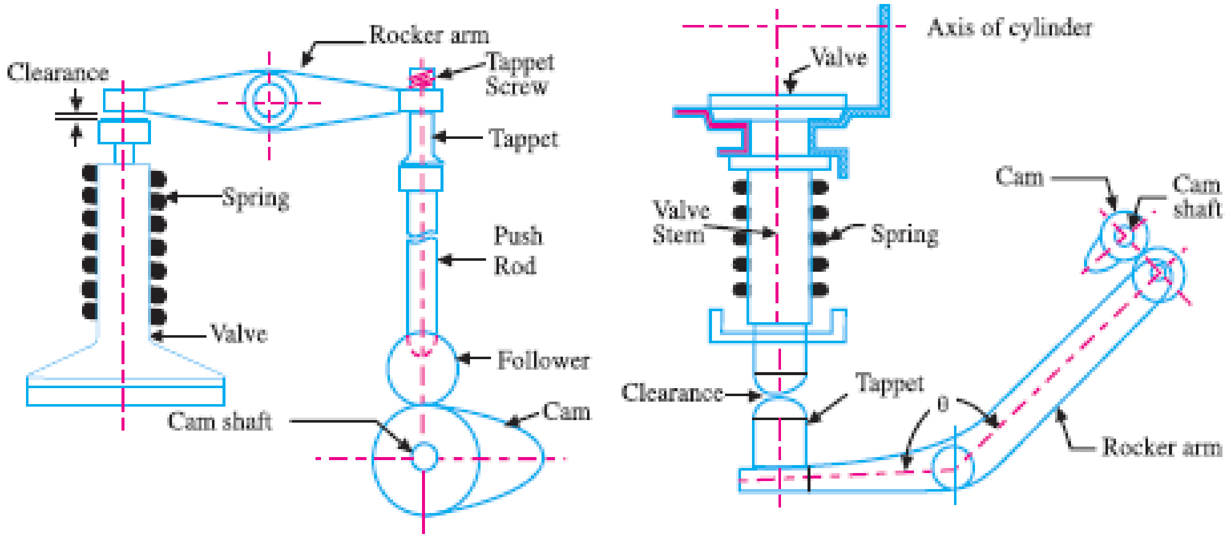


e860

- Intake valves are usually larger than exhaust valves: when the intake valve is open, air-fuel mixture is pushed into the cylinder by atmospheric pressure, in case of naturally aspirated engines. Throttling in intake valves reduces part-load efficiency.
- When the exhaust valve opens, there is still a high pressure in the engine cylinder. Hence, a smaller exhaust valve provides enough space for the high-pressure exhaust gases to get out of the cylinder.
- Some engines have 3 valves per cylinder: 2 IV and 1 EV.

Valve Gear Mechanism

The valve gear mechanism of an I.C. engine consists of those parts which actuate the inlet and exhaust valves at the required time with respect to the position of piston and crankshaft.Fig shows the valve gear arrangement for vertical engines. The main components of the mechanism are valves, rocker arm, * valve springs, **push rod, ***cam and camshaft



(a) For Vertical Engines.

(b) For Horizontal Engines.

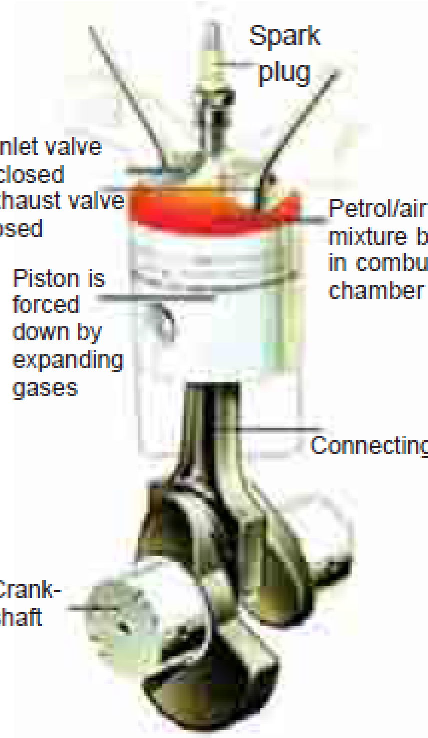
The fuel is admitted to the engine by the inlet valve and the burnt gases are escaped through the exhaust valve. In vertical engines, the cam moving on the rotating camshaft pushes the cam follower and push rod upwards, thereby transmitting the cam action to rocker arm. The camshaft is rotated by the toothed belt from the crankshaft. The rocker arm is pivoted at its centre by a fulcrum pin. When one end of the rocker arm is pushed up by the push rod, the other end moves downward. This pushes down the valve stem causing the valve to move down, thereby opening the port. When the cam follower moves over the circular portion of cam, the pushing action of the rocker arm on the valve is released and the valve returns to its seat and closes it by the action of the valve spring. In some of the modern engines, the camshaft is located at cylinder head level. In such cases, the push rod is eliminated and the roller type cam follower is made part of the rocker arm

The valves used in internal combustion engines are of the following three types ;

1. Poppet or mushroom valve ; 2. Sleeve valve ; 3. Rotary valve.

Out of these three valves, poppet valve, as shown in Fig. 32.21, is very frequently used. It consists of head, face and stem. The head and face of the valve is separated by a small margin, to aviod sharp edge of the valve and also to provide provision for the regrinding of the face. The face angle generally varies from 30° to 45°. The lower part of the stem is provided with a groove in which spring retainer lock is installed.

Since both the inlet and exhaust valves are subjected to high temperatures of 1930°C to 2200°C during the power stroke, therefore, it is necessary that the material of the valves should withstand these temperatures. Thus the material of the valves must have good heat conduction, heat resistance, corrosion resistance, wear resistance and shock resistance. It may be noted that the temperature at the inlet valve is less as compared to exhaust valve. Thus, the inlet valve is generally made of nickel chromium alloy steel and the exhaust valve (which is subjected to very high temperature of exhaust gases) is made from silchrome steel which is a special alloy of silicon and chromium.



piston and crankshaft

In designing a valve, it is required to determine the following dimensions:

(a) Size of the valve port

Let a_p = Area of the port,

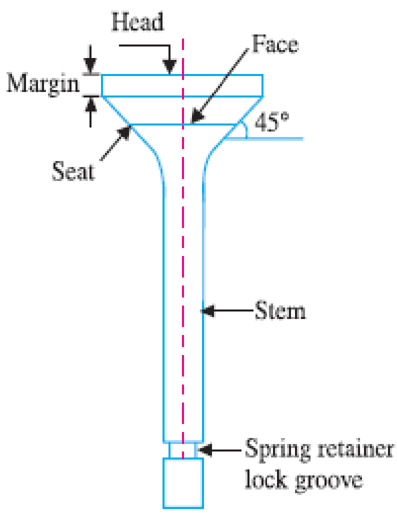
v_p = Mean velocity of gas flowing through the port,

a = Area of the piston, and

v = Mean velocity of the piston.

We know that $a_p.v_p = a.v$

$$\therefore \quad a_p = \frac{a.v}{v_p}$$



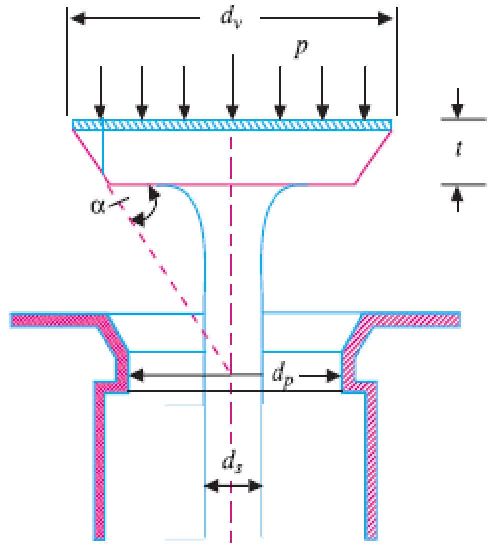


Fig. 32.21. Poppet or mushroom valve.

Fig. 32.22. Conical poppet valve in the port.

The mean velocity of the gas (v_p) may be taken from the following table.

Table 32.3. Mean velocity of the gas (v_p)

Type of engine	*Mean velocity of the gas* (v_p) m/s	
	Inlet valve	*Exhaust valve*
Low speed	33 – 40	40 – 50
High speed	80 – 90	90 – 100

Sometimes, inlet port is made 20 to 40 precent larger than exhaust port for better cylinder charging.

(b) Thickness of the valve disc

The thickness of the valve disc (t), as shown in Fig. 32.22, may be determined empirically from the following relation, *i.e.*

$$t = k.d_p \sqrt{\frac{p}{\sigma_b}}$$

where k = Constant = 0.42 for steel and 0.54 for cast iron,

d_p = Diameter of the port in mm,

p = Maximum gas pressure in N/mm^2, and

σ_b = Permissible bending stress in MPa or N/mm^2

= 50 to 60 MPa for carbon steel and 100 to 120 MPa for alloy steel.

(c) Maximum lift of the valve

h = Lift of the valve.

The lift of the valve may be obtained by equating the area across the valve seat to the area of the port. For a conical valve, as shown in Fig. 32.22, we have

$$\pi\, d_p \cdot h \cos\alpha = \frac{\pi}{4}(d_p)^2 \quad \text{or} \quad h = \frac{d_p}{4\cos\alpha}$$

where α = Angle at which the valve seat is tapered = 30° to 45°.

In case of flat headed valve, the lift of valve is given by

$$h = \frac{d_p}{4} \qquad \text{...(In this case, } \alpha = 0°)$$

The valve seats usually have the same angle as the valve seating surface. But it is preferable to make the angle of valve seat 1/2° to 1° larger than the valve angle as shown in Fig. 32.23. This results in more effective seat.

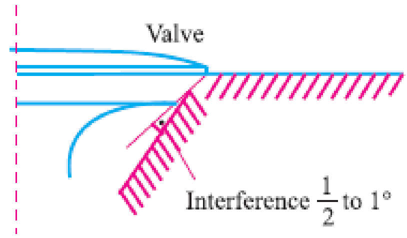


Fig. 32.23. Valve interference angle.

(d) Valve stem diameter

The valve stem diameter (d_s) is given by

$$d_s = \frac{d_p}{8} + 6.35 \text{ mm to } \frac{d_p}{8} + 11 \text{ mm}$$

Note: The valve is subjected to spring force which is taken as concentrated load at the centre. Due to this spring force (F_s), the stress in the valve (σ_t) is given by

$$\sigma_t = \frac{1.4\, F_s}{t^2}\left(1 - \frac{2d_s}{3d_p}\right)$$

Example 32.6. *The conical valve of an I.C. engine is 60 mm in diameter and is subjected to a maximum gas pressure of 4 N/mm². The safe stress in bending for the valve material is 46 MPa. The valve is made of steel for which k = 0.42. The angle at which the valve disc seat is tapered is 30°.*

Determine : 1. thickness of the valve head ; 2. stem diameter ; and 3. maximum lift of the valve.

Solution. Given : d_p = 60 mm ; p = 4 N/mm² ; σ_b = 46 MPa = 46 N/mm² ; k = 0.42 ; α = 30°

1. *Thickness of the valve head*

We know that thickness of the valve head,

$$t = k \cdot d_p \sqrt{\frac{p}{\sigma_b}} = 0.42 \times 60 \sqrt{\frac{4}{46}} = 7.43 \text{ say } 7.5 \text{ mm } \textbf{Ans.}$$

2. *Stem diameter*

We know that stem diameter,

$$d_s = \frac{d_p}{8} + 6.35 = \frac{60}{8} + 6.35 = 13.85 \text{ say } 14 \text{ mm } \textbf{Ans.}$$

3. *Maximum lift of the valve*

We know that maximum lift of the valve,

$$h = \frac{d_p}{4 \cos \alpha} = \frac{60}{4 \cos 30°} = \frac{60}{4 \times 0.866} = 17.32 \text{ say } 17.4 \text{ mm } \textbf{Ans.}$$

32.24 Rocker Arm

The * rocker arm is used to actuate the inlet and exhaust valves motion as directed by the cam and follower. It may be made of cast iron, cast steel, or malleable iron. In order to reduce inertia of the rocker arm, an *I*-section is used for the high speed engines and it may be rectangular section for low speed engines. In four stroke engines, the rocker arms for the exhaust valve is the most heavily loaded. Though the force required to operate the inlet valve is relatively small, yet it is usual practice to make the rocker

Roller followers in an engine rocker mechanism arm for the inlet valve of the same dimensions as that for exhaust valve. A typical rocker arm for operating the exhaust valve is shown in Fig. 32.24. The lever ratio a / b is generally decided by considering the space available for rocker arm. For moderate and low speed engines, a / b is equal to one. For high speed engines, the ratio a / b is taken as 1/ 1.3. The various forces acting on the rocker arm of exhaust valve are the gas load, spring force and force due to valve acceleration.

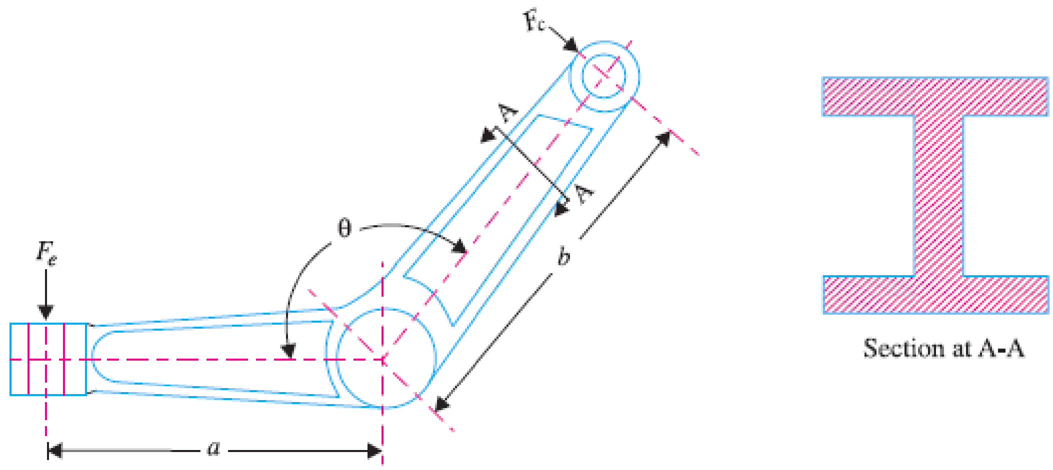


Fig. 32.24. Rocker arm for exhaust valve.

Let m_v = Mass of the valve,

d_v = Diameter of the valve head,

h = Lift of the valve,

a = Acceleration of the valve,

p_c = Cylinder pressure or back pressure when the exhust valve opens, and

p_s = Maximum suction pressure.

We know that gas load,

$$P = \text{Area of valve} \times \text{Cylinder pressure when the exhaust valve opens}$$

$$= \frac{\pi}{4} (d_v)^2 \, p_c$$

Spring force,

$$F_s = \text{Area of valve} \times \text{Maximum suction pressure}$$

$$= \frac{\pi}{4} (d_v)^2 \, p_s$$

and force due to valve acceleration,

$$F_{va} = \text{Mass of valve} \times \text{Accleration of valve}$$

$$= m_v \times a$$

∴ Maximum load on the rocker arm for exhaust valve,

$$F_e = P + F_s + F_{va}$$

It may be noted that maximum load on the rocker arm for inlet valve is

$$F_i = F_s + F_{va}$$

Since the maximum load on the rocker arm for exhaust valve is more than that of inlet valve, therefore, the rocker arm must be designed on the basis of maximum load on the rocker arm for exhaust valve, as discussed below :

1. ***Design for fulcrum pin.*** The load acting on the fulcrum pin is the total reaction (R_F) at the fulcrum point.

arm for the inlet valve of the same dimensions as that for exhaust valve. A typical rocker arm for operating the exhaust valve is shown in Fig. 32.24. The lever ratio a / b is generally decided by considering the space available for rocker arm. For moderate and low speed engines, a / b is equal to one. For high speed engines, the ratio a / b is taken as 1/ 1.3. The various forces acting on the rocker arm of exhaust valve are the gas load, spring force and force due to valve acceleration.

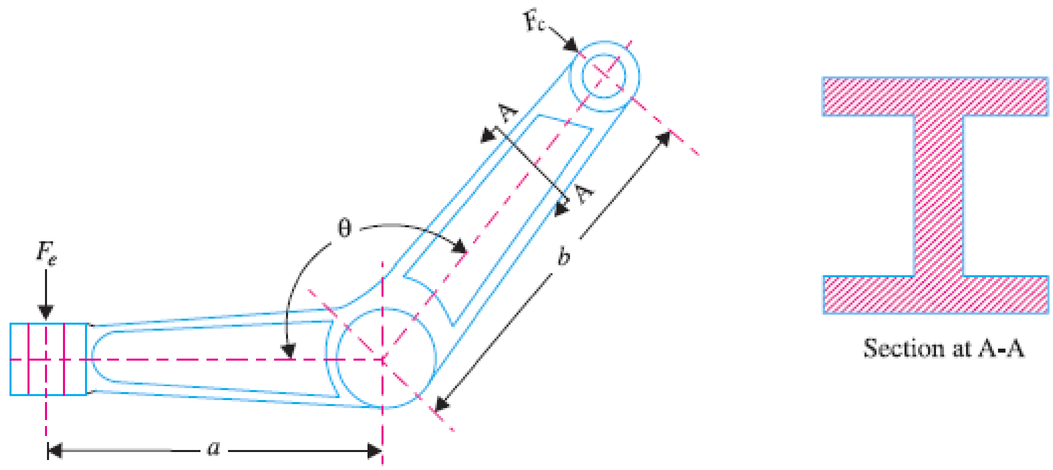


Fig. 32.24. Rocker arm for exhaust valve.

Let m_v = Mass of the valve,

d_v = Diameter of the valve head,

h = Lift of the valve,

a = Acceleration of the valve,

p_c = Cylinder pressure or back pressure when the exhust valve opens, and

p_s = Maximum suction pressure.

We know that gas load,

$$P = \text{Area of valve} \times \text{Cylinder pressure when the exhaust valve opens}$$

$$= \frac{\pi}{4} (d_v)^2 \, p_c$$

Spring force,

$$F_s = \text{Area of valve} \times \text{Maximum suction pressure}$$

$$= \frac{\pi}{4} (d_v)^2 \, p_s$$

and force due to valve acceleration,

$$F_{va} = \text{Mass of valve} \times \text{Accleration of valve}$$

$$= m_v \times a$$

∴ Maximum load on the rocker arm for exhaust valve,

$$F_e = P + F_s + F_{va}$$

It may be noted that maximum load on the rocker arm for inlet valve is

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Since the maximum load on the rocker arm for exhaust valve is more than that of inlet valve, therefore, the rocker arm must be designed on the basis of maximum load on the rocker arm for exhaust valve, as discussed below :

1. ***Design for fulcrum pin.*** The load acting on the fulcrum pin is the total reaction (R_F) at the fulcrum point.

Let d_1 = Diameter of the fulcrum pin, and

l_1 = Length of the fulcrum pin.

Considering the bearing of the fulcrum pin. We know that load on the fulcrum pin,

$$R_F = d_1 \cdot l_1 \cdot p_b$$

The ratio of l_1 / d_1 is taken as 1.25 and the bearing pressure (p_b) for ordinary lubrication is taken from 3.5 to 6 N / mm^2 and it may go upto 10.5 N/mm^2 for forced lubrication.

The pin should be checked for the induced shear stress.

The thickness of the phosphor bronze bush may be taken from 2 to 4 mm. The outside diameter of the boss at the fulcrum is usually taken twice the diameter of the fulcrum pin.

2. ***Design for forked end.*** The forked end of the rocker arm carries a roller by means of a pin. For uniform wear, the roller should revolve in the eyes. The load acting on the roller pin is F_c.

Let d_2 = Diameter of the roller pin, and

l_2 = Length of the roller pin.

Consiering the bearing of the roller pin. We know that load on the roller pin,

$$F_c = d_2 \cdot l_2 \cdot p_b$$

The ratio of l_2 / d_2 may be taken as 1.25. The roller pin should be checked for induced shear stesss.

The roller pin is fixed in eye and the thickness of each eye is taken as half the length of the roller pin.

$\therefore$ Thickness of each eye $= l_2 / 2$

The radial thickness of eye (t_3) is taken as $d_1 / 2$. Therefore overall diameter of the eye,

$$D_1 = 2\, d_1$$

The outer diameter of the roller is taken slightly larger (atleast 3 mm more) than the outer diameter of the eye.

A clearance of 1.5 mm between the roller and the fork on either side of the roller is provided.

3. ***Design for rocker arm cross-section.*** The rocker arm may be treated as a simply supported beam and loaded at the fulcrum point. We have already discussed that the rocker arm is generally of *I*-section but for low speed engines, it can be of rectangular section. Due to the load on the valve, the rocker arm is subjected to bending moment.

Let l = Effective length of each rocker arm, and

σ_b = Permissible bending stress.

We know that bending moment on the rocker arm,

$$M = F_e \times l \qquad ...(i)$$

We also know that bending moment,

$$M = \sigma_b \times Z \qquad ...(ii)$$

where Z = Section modulus.

From equations (*i*) and (*ii*), the value of Z is obtained and thus the dimensions of the section are determined.

4. ***Design for tappet.*** The tappet end of the rocker arm is made circular to receive the tappet which is a stud with a lock nut. The compressive load acting on the tappet is the maximum load on the rocker arm for the exhaust valve (F_e).

Let d_c = Core diameter of the tappet, and

σ_c = Permissible compressive stress for the material of the tappet which is made of mild steel. It may be taken as 50 MPa.

We know that load on the tappet,

$$F_e = \frac{\pi}{4} (d_c)^2 \sigma_c$$

From this expression, the core diameter of the tappet is determined. The outer or nominal diameter of the tappet (d_n) is given as

$$d_n = d_c / 0.84$$

The diameter of the circular end of the rocker arm (D_3) and its depth (t_4) is taken as twice the nominal diameter of the tappet (d_n), *i.e.*

$$D_3 = 2\, d_n\,; \text{ and } t_4 = 2\, d_n$$

5. *Design for valve spring.* The valve spring is used to provide sufficient force during the valve lifting process in order to overcome the inertia of valve gear and to keep it with the cam without bouncing. The spring is generally made from plain carbon spring steel. The total load for which the spring is designed is equal to the sum of initial load and load at full lift.

Let W_1 = Initial load on the spring

= Force on the valve tending to draw it into the cylinder on suction stroke,

W_2 = Load at full lift

= Full lift × Stiffness of spring

∴ Total load on the spring,

$$W = W_1 + W_2$$

Note : Here we are only interested in calculating the total load on the spring. The design of the valve spring is done in the similar ways as discussed for compression springs in Chapter 23 on Springs.

Design a rocker arm, and its bearings, tappet, roller and valve spring for the exhaust valve of a four stroke I.C. engine from the following data:

Diameter of the valve head = 80 mm; Lift of the valve = 25 mm; Mass of associated parts with the valve = 0.4 kg ; Angle of action of camshaft = 110° ; R. P. M. of the crankshaft = 1500.

From the probable indicator diagram, it has been observed that the greatest back pressure when the exhaust valve opens is 0.4 N/mm² and the greatest suction pressure is 0.02 N/mm² below atmosphere.

The rocker arm is to be of I-section and the effective length of each arm may be taken as 180 mm ; the angle between the two arms being 135°.

The motion of the valve may be assumed S.H.M., without dwell in fully open position.

Choose your own materials and suitable values for the stresses.

Draw fully dimensioned sketches of the valve gear.

Solution. Given : $d_v = 80$ mm ; $h = 25$ mm ; or $r = 25 / 2 = 12.5$ mm $= 0.0125$ m ; $m = 0.4$ kg ; $\alpha = 110°$; $N = 1500$ r.p.m. ; $p_c = 0.4$ N/mm² ; $p_s = 0.02$ N/mm² ; $l = 180$ mm ; $\theta = 135°$

A rocker arm for operating the exhaust valve is shown in Fig. 32.25.

First of all, let us find the various forces acting on the rocker arm of the exhaust valve.

We know that gas load on the valve,

$$P_1 = \frac{\pi}{4}(d_v)^2\, p_c = \frac{\pi}{4}(80)^2\; 0.4 = 2011 \text{ N}$$

Weight of associated parts with the valve,

$$w = m \cdot g = 0.4 \times 9.8 = 3.92 \text{ N}$$

∴ Total load on the valve,

$$P = P_1 + w = 2011 + 3.92 = 2014.92 \text{ N} \qquad \ldots(i)$$

Initial spring force considering weight of the valve,

$$F_s = \frac{\pi}{4}(d_v)^2\, p_s - w = \frac{\pi}{4}(80)^2\; 0.02 - 3.92 = 96.6 \text{ N} \qquad \ldots(ii)$$

The force due to valve acceleration (F_a) may be obtained as discussed below :

We know that speed of camshaft

$$= \frac{N}{2} = \frac{1500}{2} = 750 \text{ r.p.m.}$$

and angle turned by the camshaft per second

$$= \frac{750}{60} \times 360 = 4500 \text{ deg / s}$$

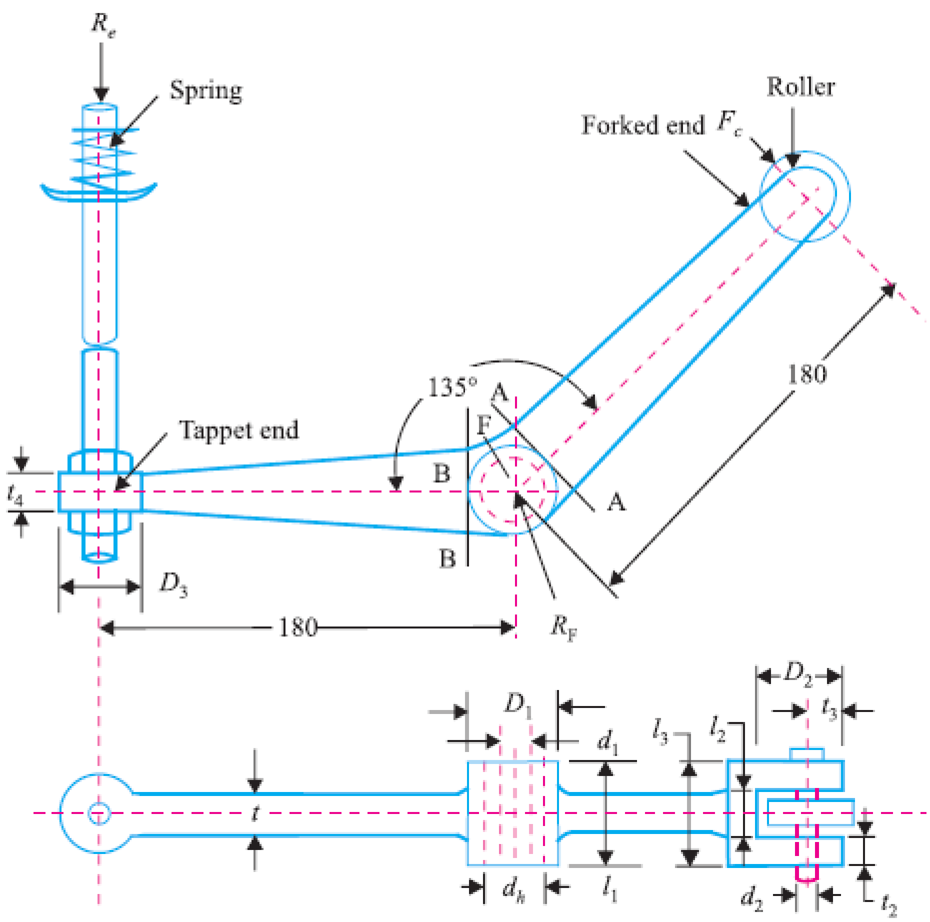


Fig. 32.25

∴ Time taken for the valve to open and close,

$$t = \frac{\text{Angle of action of cam}}{\text{Angle turned by camshaft}} = \frac{110}{4500} = 0.024 \text{ s}$$

We know that maximum acceleration of the valve

$$a = \omega^2 \cdot r = \left(\frac{2\pi}{t}\right)^2 r = \left(\frac{2\pi}{0.024}\right)^2 0.0125 = 857 \text{ m / s}^2 \quad \ldots \left(\because \omega = \frac{2\pi}{t}\right)$$

∴ Force due to valve acceleration, considering the weight of the valve,

$$F_a = m \cdot a + w = 0.4 \times 857 + 3.92 = 346.72 \text{ N} \qquad \ldots (iii)$$

and maximum load on the rocker arm for exhaust valve,

$$F_e = P + F_s + F_a = 2014.92 + 96.6 + 346.72 = 2458.24 \text{ say } 2460 \text{ N}$$

Since the length of the two arms of the rocker are equal, therefore, the load at the two ends of the arm are equal, *i.e.* $F_e = F_c = 2460$ N.

We know that reaction at the fulcrum pin *F*,

$$R_F = \sqrt{(F_e)^2 + (F_c)^2 - 2\, F_e \times F_c \times \cos\theta}$$

$$= \sqrt{(2460)^2 + (2460)^2 - 2 \times 2460 \times 2460 \times \cos 135^\circ} = 4545 \text{ N}$$

Let us now design the various parts of the rocker arm.

1. ***Design of fulcrum pin***

Let d_1 = Diameter of the fulcrum pin, and

l_1 = Length of the fulcrum pin = $1.25\, d_1$ ...(Assume)

Considering the bearing of the fulcrum pin. We know that load on the fulcrum pin (R_F),

$$4545 = d_1 \times l_1 \times p_b = d_1 \times 1.25\, d_1 \times 5 = 6.25\, (d_1)^2$$

...(For ordinary lubrication, p_b is taken as 5 N/mm^2)

$\therefore$ $(d_1)^2 = 4545 / 6.25 = 727$ or $d_1 = 26.97$ say 30 mm **Ans.**

and $l_1 = 1.25\, d_1 = 1.25 \times 30 = 37.5$ mm **Ans.**

Now let us check the average shear stress induced in the pin. Since the pin is in double shear, therefore, load on the fulcrum pin (R_F),

$$4545 = 2 \times \frac{\pi}{4}(d_1)^2 \tau = 2 \times \frac{\pi}{4}(30)^2 \tau = 1414\, \tau$$

$\therefore$ $\tau = 4545 / 1414 = 3.2$ N/mm^2 or MPa

This induced shear stress is quite safe.

Now external diameter of the boss,

$$D_1 = 2d_1 = 2 \times 30 = 60 \text{ mm}$$

Assuming a phosphor bronze bush of 3 mm thick, the internal diameter of the hole in the lever,

$$d_h = d_1 + 2 \times 3 = 30 + 6 = 36 \text{ mm}$$

Let us now check the induced bending stress for the section of the boss at the fulcrum

Bending moment at this section,

$$M = F_e \times l = 2460 \times 180 = 443 \times 10^3 \text{ N-mm}$$

Section modulus,

$$Z = \frac{\frac{1}{12} \times 37.5\,[(60)^3 - (36)^3]}{60/2} = 17\,640 \text{ mm}^3$$

∴ Induced bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{443 \times 10^3}{17\,640} = 25.1 \text{ N/mm}^2 \text{ or MPa}$$

The induced bending stress is quite safe.

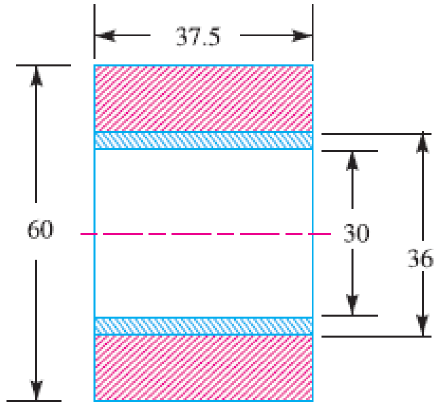


Fig. 32.26

2. *Design for forked end*

Let d_2 = Diameter of the roller pin, and

l_2 = Length of the roller pin $= 1.25\, d_1$ ...(Assume)

Considering bearing of the roller pin. We know that load on the roller pin (F_c),

$$2460 = d_2 \times l_2 \times p_b = d_2 \times 1.25\, d_2 \times 7 = 8.75\,(d_2)^2$$

... (Taking $p_b = 7$ N / mm²)

∴ $(d_2)^2 = 2460 / 8.75 = 281$ or $d_2 = 16.76$ say 18 mm **Ans.**

and $l_2 = 1.25\, d_2 = 1.25 \times 18 = 22.5$ say 24 mm **Ans.**

Let us now check the roller pin for induced shearing stress. Since the pin is in double shear, therefore, load on the roller pin (F_c),

$$2460 = 2 \times \frac{\pi}{4}(d_2)^2 \tau = 2 \times \frac{\pi}{4}(18)^2 \tau = 509\, \tau$$

∴ $\tau = 2460 / 509 = 4.83$ N/mm² or MPa

This induced shear stress is quite safe.

The roller pin is fixed in the eye and thickenss of each eye is taken as one-half the length of the roller pin.

∴ Thickness of each eye,

$$t_2 = \frac{l_2}{2} = \frac{24}{2} = 12 \text{ mm}$$

Let us now theck the induced bending stress in the roller pin. The pin is neither simply supported in fork nor rigidly fixed at the end. Therefore, the common practice is to assume the load distrubution as shown in Fig. 32.27.

The maximum bending moment will occur at $Y–Y$. Neglecting the effect of clearance, we have

Maximum bending moment at $Y – Y$,

$$M = \frac{F_c}{2}\left(\frac{l_2}{2} + \frac{t_2}{3}\right) - \frac{F_c}{2} \times \frac{l_2}{4}$$

$$= \frac{F_c}{2}\left(\frac{l_2}{2} + \frac{l_2}{6}\right) - \frac{F_c}{2} \times \frac{l_2}{4} \qquad ...(\because t_2 = l_2 / 2)$$

$$= \frac{5}{24} \times F_c \times l_2 = \frac{5}{24} \times 2460 \times 24$$

$$= 12\,300 \text{ N–mm}$$

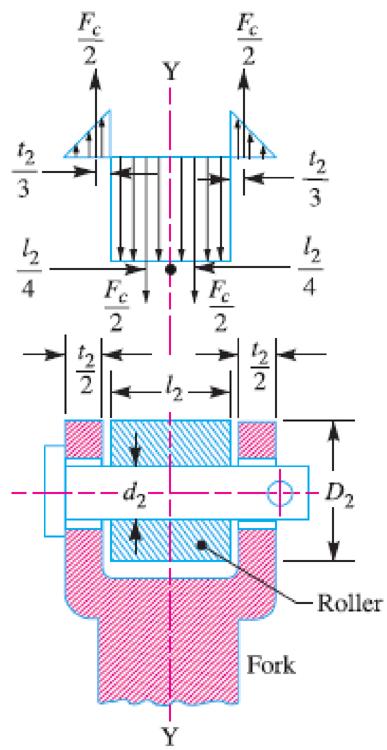


Fig. 32.27

and section modulus of the pin,

$$Z = \frac{\pi}{32}(d_2)^3 = \frac{\pi}{32}(18)^3 = 573 \text{ mm}^3$$

$\therefore$ Bending stress induced in the pin

$$= \frac{M}{Z} = \frac{12\,300}{573} = 21.5 \text{ N/mm}^2 \text{ or MPa}$$

This bending stress induced in the pin is within permissible limits.

Since the radial thickness of eye (t_3) is taken as $d_2/2$, therefore, overall diameter of the eye,

$$D_2 = 2\,d_2 = 2 \times 18 = 36 \text{ mm}$$

The outer diameter of the roller is taken slightly larger (atleast 3 mm more) than the outer diameter of the eye.

In the present case, 42 mm outer diameter of the roller will be sufficient.

Providing a clearance of 1.5 mm between the roller and the fork on either side of the roller, we have

$$l_3 = l_2 + 2 \times \frac{t_2}{2} + 2 \times 1.5$$

$$= 24 + 2 \times \frac{12}{2} + 3 = 39 \text{ mm}$$

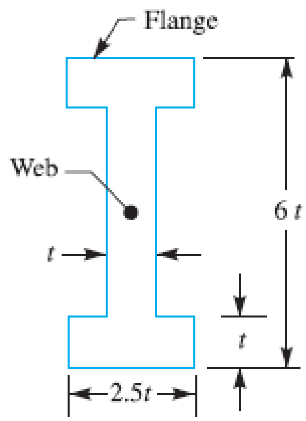


Fig. 32.28

3. *Design for rocker arm cross-section*

The cross-section of the roker arm is obtained by considering the bending of the sections just near the boss of fulcrum on both sides, such as section $A - A$ and $B - B$.

We know that maximum bending moment at $A - A$ and $B - B$.

$$M = 2460\left(180 - \frac{60}{2}\right) = 369 \times 10^3 \text{ N-mm}$$

The rocker arm is of *I*-section. Let us assume the proportions as shown in Fig. 32.28. We know that section modulus,

$$Z = \frac{\frac{1}{12}\left[2.5\,t\,(6\,t)^3 - 1.5\,t\,(4\,t)^3\right]}{6\,t/2} = \frac{37\,t^4}{3\,t} = 12.33\,t^3$$

$\therefore$ Bending stress (σ_b),

$$70 = \frac{M}{Z} = \frac{369 \times 10^3}{12.33\,t^3} = \frac{29.93 \times 10^3}{t^3}$$

$$t^3 = 29.93 \times 10^3 / 70 = 427.6 \quad \text{or} \quad t = 7.5 \text{ say } 8 \text{ mm}$$

$\therefore$ Width of flange $= 2.5\,t = 2.5 \times 8 = 20$ mm **Ans.**

Depth of web $= 4\,t = 4 \times 8 = 32$ mm **Ans.**

and depth of the section $= 6\,t = 6 \times 8 = 48$ mm **Ans.**

Normally thickness of the flange and web is constant throughout, whereas the width and depth is tapered.

4. *Design for tappet screw*

The adjustable tappet screw carries a compressive load of $F_e = 2460$ N. Assuming the screw is made of mild steel for which the compressive stress (σ_c) may be taken as 50 MPa.

Let d_c = Core diameter of the tappet screw.

We know that the load on the tappet screw (F_e),

$$2460 = \frac{\pi}{4}(d_c)^2 \sigma_c = \frac{\pi}{4}(d_c)^2\, 50 = 39.3\,(d_c)^2$$

$\therefore$ $(d_c)^2 = 2460 / 39.3 = 62.6$ or $d_c = 7.9$ say 8 mm

and outer or nominal diameter of the screw,

$$d = \frac{d_c}{0.84} = \frac{8}{0.84} = 9.52 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

We shall use 10 mm stud and it is provided with a lock nut. The diameter of the circular end of the arm (D_3) and its depth (t_4) is taken as twice the diameter of stud.

$\therefore$ $D_3 = 2 \times 10 = 20$ mm **Ans.**

and $t_4 = 2 \times 10 = 20$ mm **Ans.**

5. *Design for valve spring*

First of all, let us find the total load on the valve spring.

We know that initial load on the spring,

W_1 = Initial spring force (F_s) = 96.6 N ...(Already calculated)

and load at full lift,

W_2 = Full valve lift × Stiffiness of spring (s)

$= 25 \times 10 = 250$ N ...(Assuming s = 10 N/mm)

$\therefore$ Total load on the spring,

$$W = W_1 + W_2 = 96.6 + 250 = 346.6 \text{ N}$$

Now let us find the various dimensions for the valve spring, as discussed below:

(*a*) *Mean diameter of spring coil*

Let D = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 8 - 1}{4 \times 8 - 4} + \frac{0.615}{8} = 1.184$$

...(Assuming $C = D/d = 8$)

and maximum shear stress (τ),

$$420 = K \times \frac{8\,WC}{\pi\, d^2} = 1.184 \times \frac{8 \times 346.6 \times 8}{\pi\, d^2} = \frac{8360}{d^2}$$

...(Assuming τ = 420 MPa or N/mm^2)

$\therefore$ $d^2 = 8360 / 420 = 19.9$ or $d = 4.46$ mm

The standard size of the wire is SWG 7 having diameter (d) = 4.47 mm. **Ans.** (See Table 22.2).

$\therefore$ Mean diameter of the spring coil,

$$D = C \cdot d = 8 \times 4.47 = 35.76 \text{ mm } \textbf{Ans.}$$

and outer diameter of the spring coil,

$$D_o = D + d = 35.76 + 4.47 = 40.23 \text{ mm } \textbf{Ans.}$$

(*b*) *Number of turns of the coil*

Let n = Number of active turns of the coil.

We know that maximum compression of the spring,

$$\delta = \frac{8\, W \cdot C^3 \cdot n}{G \cdot d} \quad \text{or} \quad \frac{\delta}{W} = \frac{8\, C^3 \cdot n}{G \cdot d}$$

(*b*) *Number of turns of the coil*

Let n = Number of active turns of the coil.

We know that maximum compression of the spring,

$$\delta = \frac{8\,W \cdot C^3 \cdot n}{G \cdot d} \quad \text{or} \quad \frac{\delta}{W} = \frac{8\,C^3 \cdot n}{G \cdot d}$$

Since the stiffness of the springs, $s = W / \delta = 10$ N/mm, therefore, $\delta / W = 1/10$. Taking $G = 84 \times 10^3$ MPa or N/mm^2, we have

$$\frac{1}{10} = \frac{8 \times 8^3 \times n}{84 \times 10^3 \times 4.47} = \frac{10.9\,n}{10^3}$$

$\therefore$ $n = 10^3 / 10.9 \times 10 = 9.17$ say 10

For squared and ground ends, the total number of the turns,

$$n' = n + 2 = 10 + 2 = 12 \textbf{ Ans.}$$

(*c*) *Free length of the spring*

Since the compression produced under $W_2 = 250$ N is 25 mm (*i.e.* equal to full valve lift), therefore, maximum compression produced (δ_{max}) under the maximum load of $W = 346.6$ N is

$$\delta_{max} = \frac{25}{250} \times 346.6 = 34.66 \text{ mm}$$

We know that free length of the spring,

$$L_F = n' \cdot d + \delta_{max} + 0.15\ \delta_{max}$$
$$= 12 \times 4.47 + 34.66 + 0.15 \times 34.66 = 93.5 \text{ mm } \textbf{Ans.}$$

(*d*) *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{93.5}{12 - 1} = 8.5 \text{ mm } \textbf{Ans.}$$

Example 32. 8. *Design the various components of the valve gear mechanism for a horizontal diesel engine for the following data:*

Bore = 140 mm ; Stroke = 270 mm ; Power = 8.25 kW ; Speed = 475 r.p.m. ; Maximum gas pressure = 3.5 N/mm^2

The valve opens 33º before outer dead cerntre and closes 1º after inner dead centre. It opens and closes with constant acceleration and decleration for each half of the lift. The length of the rocker arm on either side of the fulcrum is 150 mm and the included angle is 160º. The weight of the valve is 3 N.

Solution. Given : D = 140 mm = 0.14 m ; L = 270 mm = 0.27 m ; Power = 8.25 kW = 8250 W ; N = 475 r.p.m ; p = 3.5 N/mm^2 ; l = 150 mm = 0.15 m ; θ = 160° ; w = 3 N

First of all, let us find out dimensions of the valve as discussed below :

Size of the valve port

Let d_p = Diameter of the valve port, and

$$a_p = \text{Area of the valve port} = \frac{\pi}{4}(d_p)^2$$

We know that area of the piston,

$$a = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.14)^2 = 0.0154\,\text{m}^2$$

and mean velocity of the piston,

$$v = \frac{2\,L\,N}{60} = \frac{2 \times 0.27 \times 475}{60} = 4.275 \text{ m/s}$$

From Table 32.3, let us take the mean velocity of the gas through the port (v_p) as 40 m/s.

We know that $a_p . v_p = a.v$

$$\frac{\pi}{4}(d_p)^2\, 40 = 0.0154 \times 4.275 \quad \text{or} \quad 31.42\,(d_p)^2 = 0.0658$$

$$\therefore \quad (d_p)^2 = 0.0658 / 31.42 = 2.09 \times 10^{-3} \quad \text{or} \quad d_p = 0.045 \text{ m} = 45 \text{ mm} \quad \textbf{Ans.}$$

Maximum lift of the valve

We know that maximum lift of the valve,

$$h = \frac{d_p}{4\cos\alpha} = \frac{45}{4\cos 45^\circ} = 15.9 \text{ say } 16 \text{ mm} \quad \textbf{Ans.}$$

...(Taking α = 45º)

Thickness of the valve head

We know that thickness of valve head,

$$t = k \cdot d_p \sqrt{\frac{p}{\sigma_b}} = 0.42 \times 45\sqrt{\frac{3.5}{56}} = 4.72 \text{ mm} \quad \textbf{Ans.}$$

...(Taking k = 0.42 and σ_b = 56 MPa)

Valve stem diameter

We know that valve stem diameter,

$$d_s = \frac{d_p}{8} + 6.35 \text{ mm} = \frac{45}{8} + 6.35 = 11.97 \text{ say } 12 \text{ mm} \quad \textbf{Ans.}$$

Valve head diameter

The projected width of the valve seat, for a seat angle of 45º, may be empirically taken as 0.05 d_p to 0.07 d_p. Let us take width of the valve seat as 0.06 d_p *i.e.* 0.06 × 45 = 2.7 mm.

$\therefore$ Valve head diameter, d_v = d_p + 2 × 2.7 = 45 + 5.4 = 50.4 say 51 mm **Ans.**

Now let us calculate the various forces acting on the rocker arm of exhaust valve.

We know that gas load on the valve,

$$P_1 = \frac{\pi}{4}(d_v)^2 p_c = \frac{\pi}{4}(51)^2 0.4 = 817 \text{ N} \quad \text{...(Taking } p_c = 0.4 \text{ N/mm}^2\text{)}$$

Total load on the valve, considering the weight of the valve,

$$P = P_1 + w = 817 + 3 = 820 \text{ N}$$

Initial spring force, considering the weight of the valve,

$$F_s = \frac{\pi}{4}(d_v)^2 p_s - w = \frac{\pi}{4}(51)^2 0.025 - 3 = 48 \text{ N}$$

...(Taking $p_s = 0.025$ N/mm^2)

The force due to acceleration (F_a) may be obtained as discussed below :

We know that total angle of crank for which the valve remains open

$$= 33 + 180 + 1 = 214°$$

Since the engine is a four stroke engine, therefore the camshaft angle for which the valve remains open

$$= 214 / 2 = 107°$$

Now, when the camshaft turns through 107 / 2 = 53.5°, the valve lifts by a distance of 16 mm. It may be noted that the half of this period is occupied by constant acceleration and half by constant decleration. The same process occurs when the valve closes. Therefore, the period for constant acceleration is equal to camshaft rotation of 53.5 / 2 = 26.75 ° and during this time, the valve lifts through a distance of 8 mm.

We know that speed of camshaft

$$= \frac{N}{2} = \frac{475}{2} = 237.5 \text{ r.p.m.}$$

∴ Angle turned by the camshaft per second

$$= \frac{237.5}{60} \times 360 = 1425 \text{ deg / s}$$

and time taken by the camshaft for constant acceleration,

$$t = \frac{26.75}{1425} = 0.0188 \text{ s}$$

Let a = Acceleration of the valve.

We know that $s = u \,.\, t + \frac{1}{2} a.t^2$ ... (Equation of motion)

$$8 = 0 \times t + \frac{1}{2} a\,(0.0188)^2 = 1.767 \times 10^{-4} a \quad \text{...}(\because u = o)$$

∴ $a = 8 / 1.767 \times 10^{-4} = 45\ 274 \text{ mm / s}^2 = 45.274 \text{ m / s}^2$

and force due to valve acceleration, considering the weight of the valve,

$$F_a = m \cdot a + w = \frac{3}{9.81} \times 45.274 + 3 = 16.84 \text{ N} \quad \text{... }(\because m = w/g)$$

We know that the maximum load on the rocker arm for exhaust valve,

$$F_e = P + F_s + F_a = 820 + 48 + 16.84 = 884.84 \text{ say } 885 \text{ N}$$

Since the length of the two arms of the rocker are equal, therefore, load at the two ends of the arm are equal, *i.e.* $F_e = F_c = 885$ N.

We know that reaction at the fulcrum pin F,

$$R_F = \sqrt{(F_e)^2 + (F_c)^2 - 2F_e \times F_c \times \cos\theta}$$

$$= \sqrt{(885)^2 + (885)^2 - 2 \times 885 \times 885 \times \cos 160^\circ} = 1743 \text{ N}$$

The rocker arm is shown in Fig. 32.29. We shall now design the various parts of rocker arm as discussed below:

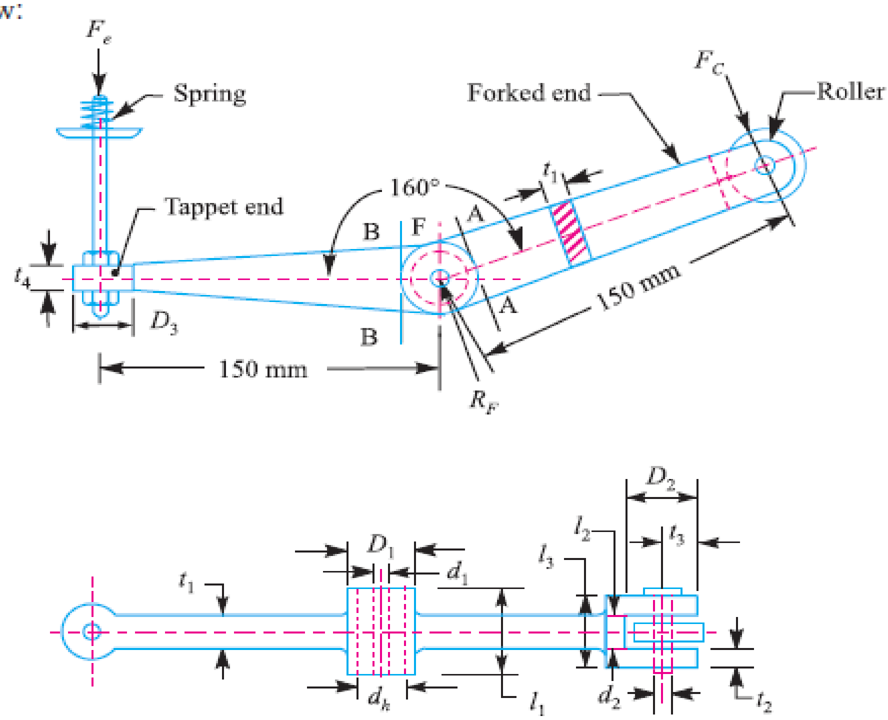


Fig. 32.29

1. *Design of fulcrum pin*

Let d_1 = Diameter of the fulcrum pin, and

l_1 = Length of the fulcrum pin = 1.25 d_1 ... (Assume)

Considering the bearing of the fulcrum pin. We know that load on the fulcrum pin (R_F),

$$1743 = d_1 \times l_1 \times p_b = d_1 \times 1.25\, d_1 \times 5 = 6.25\,(d_1)^2$$

... (For ordinary lubrication, p_b is taken as 5 N/mm²)

$\therefore$ $(d_1)^2 = 1743 / 6.25 = 279$ or $d_1 = 16.7$ say 17 mm

and $l_1 = 1.25\, d_1 = 1.25 \times 17 = 21.25$ say 22 mm

Now let us check the average shear stress induced in the pin. Since the pin is in double shear, therefore, load on the fulcrum pin (R_F),

$$1743 = 2 \times \frac{\pi}{4}(d_1)^2 \tau = 2 \times \frac{\pi}{4}(17)^2 \tau = 454\,\tau$$

$\therefore$ $\tau = 1743 / 454 = 3.84$ N/mm² or MPa

This induced shear stress is quite safe.

Now external diameter of the boss,

$$D_1 = 2d_1 = 2 \times 17 = 34 \text{ mm}$$

Assuming a phosphor bronze bush of 3 mm thick, the internal diameter of the hole in the lever,

$$d_h = d_1 + 2 \times 3 = 17 + 6 = 23 \text{ mm}$$

Now, let us check the induced bending stress for the section of the boss at the fulcrum which is shown in Fig. 32.30.

Bending moment at this section,

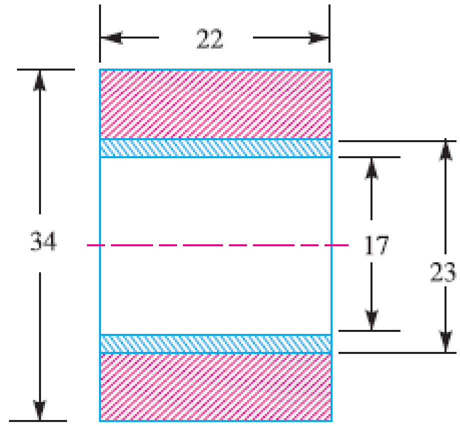
$$M = F_e \times l = 885 \times 150 \text{ N-mm}$$
$$= 132\ 750 \text{ N-mm}$$

Section modulus,

$$Z = \frac{\frac{1}{12} \times 22\ [(34)^3 - (23)^3]}{34/2} = 2927 \text{ mm}^3$$

$\therefore$ Induced bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{132\ 750}{2927} = 45.3 \text{ N/mm}^2 \text{ or MPa}$$



All dimensions in mm

Fig. 32.30

The induced bending stress is quite safe.

2. Design for forked end

Let d_2 = Diameter of the roller pin, and

l_2 = Length of the roller pin = $1.25\ d_2$ ...(Assume)

Considering bearing of the roller pin. We know that load on the roller pin (F_c),

$$885 = d_2 \times l_2 \times p_b = d_2 \times 1.25\ d_2 \times 7 = 8.75\ (d_2)^2$$

...(Taking p_b = 7 N/mm²)

$\therefore$ $(d_2)^2 = 885 / 8.75 = 101.14$ or $d_2 = 10.06$ say 11 mm **Ans.**

and $l_2 = 1.25\ d_2 = 1.25 \times 11 = 13.75$ say 14 mm **Ans.**

Let us now check the roller pin for induced shearing stress. Since the pin is in double shear, therefore, load on the roller pin (F_c),

$$885 = 2 \times \frac{\pi}{4}(d_2)^2 \tau = 2 \times \frac{\pi}{4}(11)^2 \tau = 190\ \tau$$

$\therefore \quad \tau = 885 / 190 = 4.66$ N/mm² or MPa

This induced shear stress is quite safe.

The roller pin is fixed in the eye and thickness of each eye is taken as one-half the length of the roller pin.

$\therefore$ Thickness of each eye,

$$t_2 = \frac{l_2}{2} = \frac{14}{2} = 7 \text{ mm}$$

Let us now check the induced bending stress in the roller pin. The pin is neither simply supported in fork nor rigidly fixed at the end. Therefore, the common practice is to assume the load distribution as shown in Fig. 32.31.

The maximum bending moment will occur at Y– Y. Neglecting the effect of clearance, we have

Maximum bending moment at Y–Y,

$$M = \frac{F_c}{2}\left(\frac{l_2}{2} + \frac{t_2}{3}\right) - \frac{F_c}{2} \times \frac{l_2}{4}$$

$$= \frac{F_c}{2}\left(\frac{l_2}{2} + \frac{l_2}{6}\right) - \frac{F_c}{2} \times \frac{l_2}{4} \quad ...(\because t_2 = l_2/2)$$

$$= \frac{5}{24} \times F_c \times l_2$$

$$= \frac{5}{24} \times 885 \times 14 = 2581 \text{ N-mm}$$

and section modulus of the pin,

$$Z = \frac{\pi}{32}(d_2)^3 = \frac{\pi}{32}(11)^3 = 131 \text{ mm}^3$$

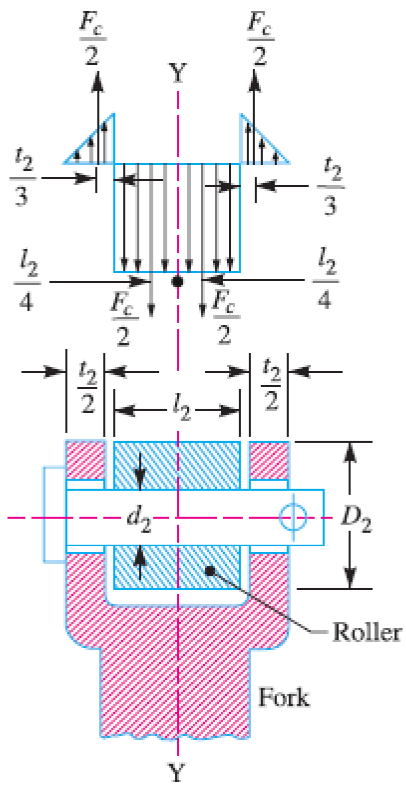


Fig. 32.31

$\therefore$ Bending stress induced in the pin

$$= \frac{M}{Z} = \frac{2581}{131} = 19.7 \text{ N/mm}^2 \text{ or MPa}$$

This bending stress induced in the pin is within permissible limits.

Since the radial thickness of eye (t_3) is taken as $d_2 / 2$, therefore, overall diameter of the eye,

$$D_2 = 2\ d_2 = 2 \times 11 = 22 \text{ mm}$$

The outer diameter of the roller is taken slightly larger (at least 3 mm more) than the outer diameter of the eye. In the present case, 28 mm outer diameter of the roller will be sufficient.

Providing a clearance of 1.5 mm between the roller and the fork on either side of the roller, we have

$$l_3 = l_2 + 2 \times \frac{t_2}{2} + 2 \times 1.5 = 14 + 2 \times \frac{7}{2} + 3 = 24 \text{ mm}$$

3. Design for rocker arm cross-section

Since the engine is a slow speed engine, therefore, a rectangular section may be selected for the rocker arm. The cross-section of the rocker arm is obtained by considering the bending of the sections just near the boss of fulcrum on both sides, such as section A–A and B– B.

Let t_1 = Thickness of the rocker arm which is uniform throughout.

B = Width or depth of the rocker arm which varies from boss diameter of fulcrum to outside diameter of the eye (for the forked end side) and from boss diameter of fulcrum to thickness t_3 (for the tappet or stud end side).

Now bending moment on section $A - A$ and $B - B$,

$$M = 885\left(150 - \frac{34}{2}\right) = 117\ 705 \text{ N-mm}$$

and section modulus at $A - A$ and $B - B$,

$$Z = \frac{1}{6} \times t_1 \cdot B^2 = \frac{1}{6} \times t_1\ (D_1)^2 = \frac{1}{6} \times t_1\ (34)^2 = 193\ t_1$$

...(At sections A–A and B–B, $B = D$)

We know that bending stress (σ_b),

$$70 = \frac{M}{Z} = \frac{117\ 705}{193\ t_1} \qquad \text{...(Taking } \sigma_b = 70 \text{ MPa or N/mm}^2\text{)}$$

$\therefore$ $t_1 = 117\ 705 / 193 \times 70 = 8.7$ say 10 mm **Ans.**

4. *Design for tappet screw*

The adjustable tappet screw carries a compressive load of $F_e = 885$ N. Assuming the screw to be made of mild steel for which the compressive stress (σ_c) may be taken as 50 MPa.

Let d_c = Core diameter of the tappet screw.

We know that load on the tappet screw (F_e),

$$885 = \frac{\pi}{4}\ (d_c^{\ 2})\ \sigma_c = \frac{\pi}{4}\ (d_c)^2\ 50 = 39.3\ (d_c)^2$$

$\therefore$ $(d_c)^2 = 885 / 39.3 = 22.5$ or $d_c = 4.74$ say 5 mm **Ans.**

and outer or nominal diameter of the screw,

$$d = \frac{d_c}{0.84} = \frac{5}{0.8} = 6.25 \text{ say } 6.5 \text{ mm } \textbf{Ans.}$$

We shall use 6.5 mm stud and it is provided with a lock nut. The diameter of the circular end of the arm (D_3) and its depth (t_4) is taken as twice the diameter of stud.

$\therefore$ $D_3 = 2 \times 6.5 = 13$ mm **Ans.**

and $t_4 = 2 \times 6.5 = 13$ mm **Ans.**

5. *Design for valve spring*

First of all, let us find the total load on the valve spring.

We know that intial load on the spring,

W_1 = Initial spring force (F_s) = 48 N ...(Already calculated)

and load at full lift, W_2 = Full valve lift × Stiffness of spring (s)

$= 16 \times 8 = 128$ N ...(Taking $s = 8$ N/mm)

$\therefore$ Total load on the spring,

$$W = W_1 + W_2 = 48 + 128 = 176 \text{ N}$$

Now let us find the various dimensions for the valve spring as discussed below:

(a) Mean diameter of the spring coil

Let D = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

...(Assuming $C = D/d = 6$)

and maximum shear stress (τ),

$$420 = K \times \frac{8\,WC}{\pi\,d^2} = 1.2525 \times \frac{8 \times 176 \times 6}{\pi\,d^2} = \frac{3368}{d^2}$$

$\therefore$ $d^2 = 3368 / 420 = 8.02$ or $d = 2.83$ mm

The standard size of the wire is SWG 11 having a diameter (d) = 2.946 mm **Ans.** (see Table 22.2)

$\therefore$ Mean diameter of spring coil,

$$D = C \cdot d = 6 \times 2.946 = 17.676 \text{ mm} \quad \textbf{Ans.}$$

and outer diameter of the spring coil,

$$D_o = D + d = 17.676 + 2.946 = 20.622 \text{ mm} \quad \textbf{Ans.}$$

(*b*) *Number of turns of the coil*

Let n = Number of turns of the coil,

We know that maximum compression of the spring.

$$\delta = \frac{8\,W \cdot C^3 \cdot n}{G \cdot d} \quad \text{or} \quad \frac{\delta}{W} = \frac{8\,C^3 \cdot n}{G \cdot d}$$

Since the stiffness of the spring, $s = W/\delta = 8$ N / mm, therefore $\delta / W = 1/8$. Taking $G = 84 \times 10^3$ MPa or N/mm^2, we have

$$\frac{1}{8} = \frac{8 \times 6^3 \times n}{84 \times 10^3 \times 2.946} = \frac{6.98\, n}{10^3}$$

$$\therefore \quad n = 10^3 / 8 \times 6.98 = 17.9 \text{ say } 18$$

For squared and ground ends, the total number of turns,

$$n' = n + 2 = 18 + 2 = 20 \text{ **Ans.**}$$

(*c*) *Free length of the spring*

Since the compression produced under $W_2 = 128$ N is 16 mm, therefore, maximum compression produced under the maximum load of $W = 176$ N is

$$\delta_{max} = \frac{16}{128} \times 176 = 22 \text{ mm}$$

We know that free length of the spring,

$$L_F = n'.\, d + \delta_{max} + 0.15\, \delta_{max}$$
$$= 20 \times 2.946 + 22 + 0.15 \times 22 = 84.22 \text{ say } 85 \text{ mm}$$ **Ans.**

(*d*) *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{85}{20 - 1} = 4.47 \text{ mm}$$ **Ans.**

Design of cam

The cam is forged as one piece with the camshaft. It is designed as discussed below :

The diameter of camshaft (D') is taken empirically as

$$D' = 0.16 \times \text{Cylinder bore} + 12.7 \text{ mm}$$
$$= 0.16 \times 140 + 12.7 = 35.1 \text{ say } 36 \text{ mm}$$

The base circle diameter is about 3 mm greater than the camshaft diameter.

$\therefore$ Base circle diameter $= 36 + 3 = 39$ say 40 mm

The width of cam is taken equal to the width of roller, *i.e.* 14 mm.

The width of cam (w') is also taken empirically as

$$w' = 0.09 \times \text{Cylinder bore} + 6 \text{ mm} = 0.09 \times 140 + 6 = 18.6 \text{ mm}$$

Let us take the width of cam as 18 mm.

Now the *cam is drawn according to the procedure given below :

First of all, the displacement diagram, as shown in Fig. 32.32, is drawn as discussed in the following steps :

1. Draw a horizontal line *ANM* such that *AN* represents the angular displacement when valve opens (*i.e.* 53.5°) to some suitable scale. The line *NM* represents the angular displacement of the cam when valve closes (*i.e.* 53.5°).
2. Divide *AN* and *NM* into any number of equal even parts (say six).
3. Draw vertical lines through points 0, 1, 2, 3 etc. equal to the lift of valve *i.e.* 16 mm.
4. Divide the vertical lines 3 – *f* and 3′ – *f*′ into six equal parts as shown by points *a, b, c* ... and *a*′, *b*′, *c*′ .... in Fig. 32.32.
5. Since the valve moves with equal uniform acceleration and decleration for each half of the lift, therefore, valve displacement diagram for opening and closing of valve consists of double parabola.

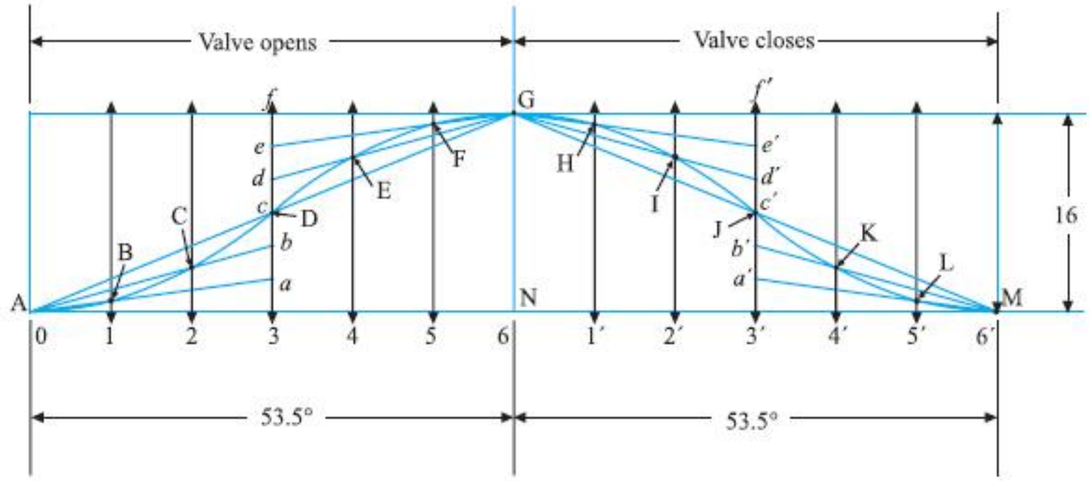


Fig. 32.32. Displacement diagram.

6. Join *Aa*, *Ab*, *Ac* intersecting the vertical lines through 1, 2, 3 at *B*, *C*, *D* respectively.
7. Join the points *B*, *C*, *D* with a smooth curve. This is the required parabola for the half of valve opening. Similarly other curves may be drawn as shown in Fig. 32.32.
8. The curve *A*, *B*, *C*, ..., *G*, *K*, *L*, *M* is the required displacement diagram.

Gears keyed to camshafts

Now the profile of the cam, as shown in Fig. 32.32, is drawn as discussed in the following steps:

1. Draw a base circle with centre *O* and diameter equal 40 mm (radius = 40/2 = 20 mm)
2. Draw a prime circle with centre *O* and radius, OA = Min. radius of cam + $\frac{1}{2}$ Diameter of roller $= 20 + \frac{1}{2} \times 28$

$$= 20 + 14 = 34 \text{ mm}$$

3. Draw angle $AOG = 53.5°$ to represent opening of valve and angle $GOM = 53.5°$ to represent closing of valve.
4. Divide the angular displacement of the cam during opening and closing of the valve (*i.e.* angle *AOG* and *GOM*) into same number of equal even parts as in displacement diagram.
5. Join the points 1, 2, 3, etc. with the centre *O* and produce the lines beyond prime circle as shown in Fig. 32.33.
6. Set off points 1*B*, 2*C*, 3*D*, etc. equal to the displacements from displacement diagram.
7. Join the points *A*, *B*, *C*, ...*L*, *M*, *A*. The curve drawn through these points is known as ***pitch curve.***
8. From the points *A*, *B*, *C*, ...*K*, *L*, draw circles of radius equal to the radius of the roller.

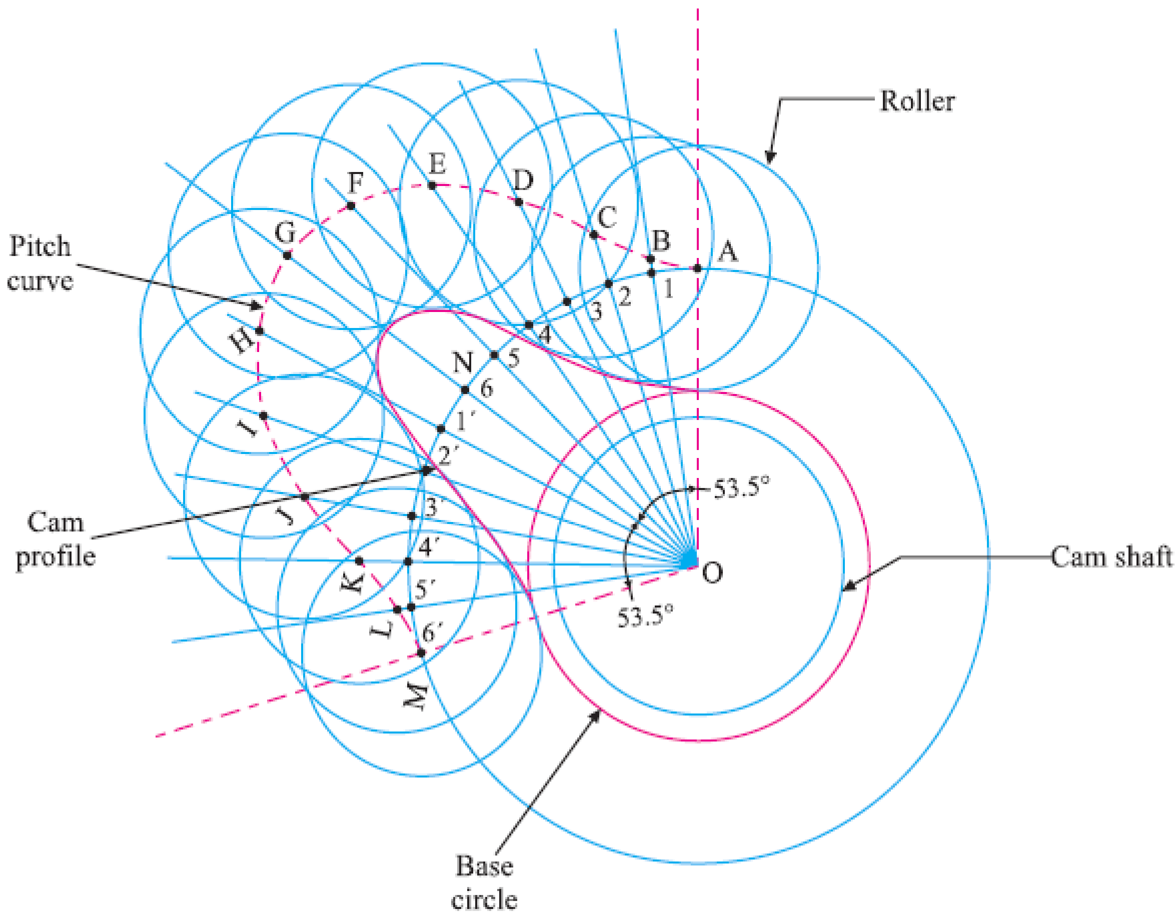


Fig. 32.33

9. Join the bottoms of the circle with a smooth curve as shown in Fig. 32.33. The is the required profile of cam.